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Effect of porous lining on the flow between two concentric rotating cylinders

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Abstract. The effect of non-erodible porous lining on the flow between two concentric rotating cylinders is investigated using Beavers and Joseph slip boundary condition. It is shown that the shearing stress at the walls increases with the porous lining thickness parameter ε.

Keywords. Porous lining; rotating cylinders; shearing stress.

1. Introduction

The control of shearing stress is important in the design of rotating machinery (like totally enclosed —fan-cooled motors and lubrication industry) in which the centrifugal force plays a major role. To achieve this, following the rectangular geometry model of Channabasappa *et al* [3], we consider the flow between two concentric rotating cylinders with non-erodible porous lining on the inner wall of the outer cylinder. The velocity field, using Beavers and Joseph [1] slip boundary condition (hereafter called the BJ condition), is determined and it is shown that the velocity increases with the porous lining thickness parameter ε. Shearing stress at the walls is calculated for different values of ε and it is shown that it increases with ε.

2. Formulation of the problem

The physical model illustrating the problem under consideration is shown in figure 1.

We consider an axi-symmetric steady incompressible viscous flow in an annulus between two concentric rotating cylinders. The inner impermeable cylinder of radius a rotates with an angular velocity ω, while the outer cylinder of radius b ($> a$) rotating with an angular velocity Ω has a non-erodible porous lining of thickness h on its inner wall. The flow in the annulus (called zone 1) is governed by the usual

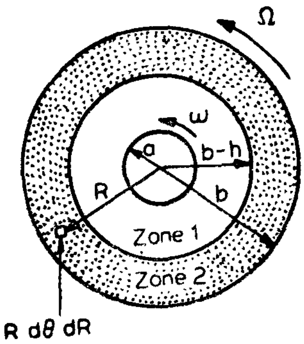

Figure 1. Physical model.

Navier–Stokes equations and that in the porous medium (called zone 2) by the Darcy law. The basic equations of the flow in zone 1, assuming that the flow is caused by the rotation of the cylinders, in cylindrical coordinates are [2]

$$\rho\left(U\frac{dU}{dR}-\frac{V^2}{R}\right)=-\frac{dP}{dR}+\mu\left(\frac{d^2U}{dR^2}+\frac{1}{R}\frac{dU}{dR}-\frac{U}{R^2}\right) \tag{1}$$

$$\rho\left(U\frac{dV}{dR}+\frac{UV}{R}\right)=\mu\left(\frac{d^2V}{dR^2}+\frac{1}{R}\frac{dV}{dR}-\frac{V}{R^2}\right). \tag{2}$$

The equation of continuity is

$$\frac{dU}{dR}+\frac{U}{R}=0, \tag{3}$$

where U and V are the radial and azimuthal components of the velocity and R the radial distance. The boundary conditions are

$$U=0 \text{ at } R=a \text{ and } R=b-h \tag{4}$$

$$V=a\omega \text{ at } R=a \tag{5}$$

$$V=V_B \text{ at } R=b-h \tag{6}$$

where V_B is obtained by using the BJ condition

$$\left.\frac{dV}{dR}\right|_{R=b-h}=\frac{\alpha}{\sqrt{K}}\left\{V_B-Q\Big|_{b-h}\right\}, \tag{7}$$

where Q is the Darcy velocity in zone 2 given by

$$Q=R\Omega+E. \tag{8}$$

Here $R\Omega$ is the velocity in the porous medium due to the rotation of the porous medium itself with an angular velocity Ω and E is given by

$$E=\frac{K}{\mu}\frac{\int_0^{2\pi}\int_{b-h}^{b}\rho R\Omega^2(R\,d\theta\,dR)}{\int_0^{2\pi}\int_{b-h}^{b}R\,d\theta\,dR},$$

$$=\frac{K}{\mu}\frac{2}{3}\rho\Omega^2\left\{\frac{3b^2-3bh+h^2}{2b-h}\right\}. \tag{9}$$

The quantities K and α in (7) are respectively the permeability and the slip parameter of the porous material. For a given porous material K can be determined experimentally by using the usual permeameter apparatus, while α, since it is independent of the geometry of flow, can be determined by using the experimental set up of Beavers and Joseph [1].

We note that the equation of continuity (3) implies

$$U = C/R, \tag{10}$$

where C is a constant. Since there is no suction or injection normal to the walls, this C has to be zero to satisfy the no slip boundary condition (4).

Therefore $U \equiv 0$ everywhere. (11)

In view of (11), the solution of (2) satisfying the boundary conditions (5) and (6) is

$$v = \frac{1}{r\{\lambda^2 - (1-\varepsilon)^2\}} [\lambda^2 \beta\{r^2 - (1-\varepsilon)^2\} + v_B (1-\varepsilon)(\lambda^2 - r^2)] \tag{12}$$

where
$$v_B = \frac{(1-\varepsilon)\{\lambda^2 - (1-\varepsilon)^2\}\{3\alpha\sigma^2(\varepsilon^2 - 3\varepsilon + 2) + 2\alpha\,\mathrm{Re}\,(\varepsilon^2 - 3\varepsilon + 3)\} + 6\sigma\lambda^2\beta(\varepsilon^2 - 3\varepsilon + 2)}{3\alpha\sigma^2(\varepsilon^2 - 3\varepsilon + 2)\{\lambda^2 - (1-\varepsilon)^2\} + 3\sigma(2-\varepsilon)\{\lambda^2 + (1-\varepsilon)^2\}} \tag{13}$$

and
$$[v, r, \lambda, \sigma, \beta, \varepsilon, \mathrm{Re}, p, \tau] = \left[\frac{V}{b\Omega}, \frac{R}{b}, \frac{a}{b}, \frac{b}{\sqrt{K}}, \frac{\omega}{\Omega}, \frac{h}{b}, \frac{b^2\Omega}{\nu}, \frac{P}{\rho b^2 \Omega^2}, \frac{T}{\mu\Omega}\right]. \tag{14}$$

Using (11) in (1), we get

$$\frac{v^2}{r} = \frac{dp}{dr} \tag{15}$$

which gives the radial pressure.

The shearing stress can be computed from (12), using

$$\tau = r\,(d/dr)\,(v/r) \tag{16}$$

and is of the form

$$\tau = \frac{2\lambda^2(1-\varepsilon)\{\beta(1-\varepsilon) - v_B\}}{r^2\{\lambda^2 - (1-\varepsilon)^2\}}. \tag{17}$$

Thus the shearing stresses at the inner and the outer walls are respectively given by

$$\tau_{pi} = \frac{2(1-\varepsilon)\{\beta(1-\varepsilon) - v_B\}}{\{\lambda^2 - (1-\varepsilon)^2\}} \tag{18}$$

$$\tau_{po} = \frac{2\lambda^2\{\beta(1-\varepsilon) - v_B\}}{(1-\varepsilon)\{\lambda^2 - (1-\varepsilon)^2\}} \tag{19}$$

These in the limit $\varepsilon \to 0$ and $\sigma \to \infty$ reduce to the results of impermeable case [4]

$$\tau_{ri} = \frac{2(\beta - 1)}{\lambda^2 - 1} \tag{20}$$

$$\tau_{ra} = \frac{2\lambda^2(\beta - 1)}{\lambda^2 - 1}\,. \tag{21}$$

3. Conclusions

The velocity field given by (12) and the shearing stresses given by (18) and (19) are numerically computed for certain combinations of the parameters and the results are presented graphically in figures 2 and 3.

From figure 2 it is clear that the velocity increases with the porous lining thickness ε. Figure 3 represents the effect of ε on shearing stress, and from this we conclude that (i) the shearing stresses (at the walls) with porous lining are always greater than the corresponding ones for the impermeable case, (ii) the shearing stresses increase with the thickness of the porous lining.

Typical behaviours are observed for other parameters.

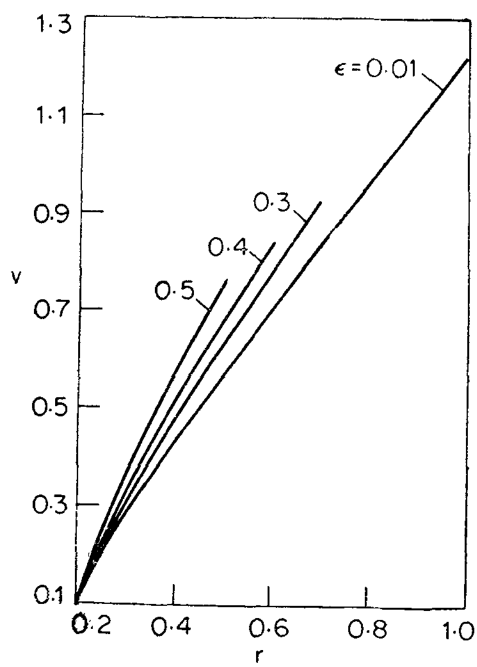

Figure 2. Variation of v with r for $\lambda = 0{\cdot}2$; $\beta = 0{\cdot}5$; $\sigma = 100$; Re = 1000.

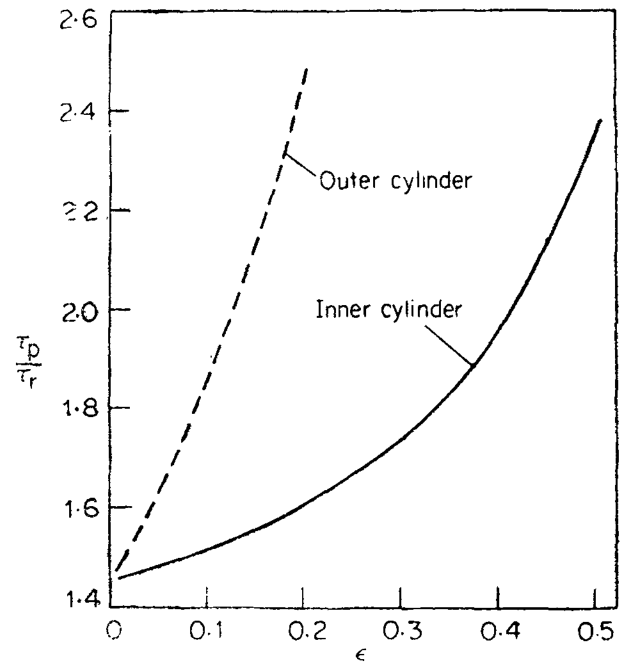

Figure 3. Variation of τ with ε for $\lambda = 0{\cdot}2$; $\beta = 0{\cdot}5$; $\sigma = 100$; Re $= 1000$.

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