DE GRUYTER International Journal of Emerging Electric Power Systems. 2017; 20160201

Tukaram Moger[1] / Thukaram Dhadbanjan[2]

Evaluation of Reactive Power Support and Loss Allocation in a Pool Based Competitive Electricity Market

[1] Department of Electrical and Electronics Engineering, National Institute of Technology Karnataka, Surathkal, Mangalore-575025, Karnataka, India, E-mail: tukaramoger@ee.iisc.ernet.in

[2] Department of Electrical Engineering, Indian Institute of Science, Bangalore-560012, Karnataka, India, E-mail: dtram@ee.iisc.ernet.in

Abstract:
This paper presents a new approach using modified Y-bus matrix to compute the reactive power support and loss allocation in a pool based competitive electricity market. The inherent characteristic of the reactive power in system operation is properly addressed in the paper. A detailed case study on a 11-bus equivalent system is carried out to illustrate the effectiveness of the proposed approach. It is also tested on a large 259-bus equivalent system of Indian western region power grid. A comparison is also made with other existing approaches in the literature to highlight the features of the proposed approach. Simulation results show that the reactive power support and loss allocation from the proposed approach is carried out in a systematic manner which takes into consideration the power demand and the relative location of the nodes in the network.

Keywords: reactive power support allocation, reactive power loss allocation, power system deregulation, modified Y-bus matrix

DOI: 10.1515/ijeeps-2016-0201

1 Introduction

In recent years, the electric power industry around the world is changing continuously due to transformation from regulated market structure to deregulated market structure. As a result, several new issues and challenges have emerged. One of the main issues in a deregulated power system is to support reactive power for maintaining the system voltage profile with an acceptable margin of security and reliability required for system operation. Not only is the reactive power necessary to operate the transmission system reliably, but it can also substantially improve the efficiency with which real power is delivered to the consumers. Inadequate reactive power has led to voltage collapses and has been a cause of several recent major power outages worldwide [1]. Such activity can be considered under the separate market called reactive power ancillary service market. Power transfer in the transmission network causes active and reactive power losses due to the resistance and reactance of the network. It is well known that the reactive power loss also depends on the active power flows and even a load with unity power factor can cause reactive power to flow in the network [2]. With increased level of competition in a deregulated open access market, it is more important to know the amount of reactive power contributed by various reactive sources. It is also necessary to know the amount of reactive power consumed by the loads and how much a particular load is responsible for contributing to the reactive power loss in the system. Especially, under heavily loaded conditions, the amount of reactive power loss may exceed the total reactive power demand of the system. Hence, reactive power loss should be considered in the evaluation of the system's total reactive power requirement. Developing a fair and adequate method of determining the reactive power support and loss allocation may give correct economic signals to market participants and system operator regarding the system reactive power issues. It may help the market participants to make appropriate and efficient investment on reactive sources (reactive power ancillary service market). This can offer system operator more tools and can strengthen the system security. Moreover, the reactive power support and loss allocation scheme must be transparent, and take into consideration the relative location of the nodes within the network.

Several methodologies/approaches have been proposed in the literature for allocating the system loss to generators and/or loads in a deregulated power system based on different assumptions and approximations. These can be classified under pro-rata methods [3], incremental transmission loss methods [4], methods based

---

Tukaram Moger is the corresponding author.

on proportional sharing principles (PSP) [5–10], circuit based methods [11–14] and different other approaches [15]. The merits and demerits of the above mentioned methods are also reported in the literature [3–15]. Among the circuit based allocation methods [11–14], the Z-bus matrix and modified Y-bus matrix methods are more popular. These methods can integrate the network characteristics in terms of network equation directly. Since it is based on a solved power flow, it can consider system non-linearity accurately and all its computations are based on system admittance matrix. Hence, circuit based methods are very efficient and suitable for use in real system. An approach based on modified Y-bus matrix is proposed in Chu et al. [12] for allocating the reactive power support to load buses from the generator reactive sources without explicitly considering the network reactive sources. Further, the aspects of reactive power loss is also not considered during the allocation process. The same concept is extended in De and Goswami [13] considering the network reactive sources. In Thukaram and Vyjayanthi [14], proposed relative electrical distance concept to determine the reactive power contributions from source to load. The reactive power loss is allocated to load buses based on the relative electrical distance concept. This work is an extension to the approach presented in Moger and Dhadbanjan [16]. As compared to [16], the following aspects are discussed in more details: (1) The issues associated with reactive power support and loss allocation are discussed, and significant observations are reported; (2) The detailed evaluation of reactive power support and loss allocation under various system operating/loading conditions, and its impact on system reactive power issues are presented; (3) To test the effectiveness of the proposed approach, a comparative analysis with one of the existing circuit based approach [12] is carried out; and (4) Further, the results on 259-bus equivalent system of Indian western region power grid with additional discussions are presented.

This paper presents a new approach using modified Y-bus matrix to compute the reactive power contribution from various reactive sources to meet the reactive load demand and losses. Further, the allocation of reactive power loss to load/sink buses is also computed. In order to account for the reactive power support produced by the line charging capacitances, the equivalent transmission line model is considered by integrating the shunt part of the line into nearby buses. The amount of reactive power support produced by the line charging capacitance depends on the operating condition of the system, which may be less during the peak load. However, for proper accounting of reactive sources of the network under open access deregulated market, it is essential to consider these in the evaluation of the total system's reactive power requirements. A detailed case studies on a 11-bus equivalent system, which is a part of Indian southern region power grid is carried out under different loading conditions. The loading conditions on the system are simulated by adopting load scale factor (LSF), which is a ratio between the actual system loading and the base case loading. A large 259-bus equivalent system of Indian western region power grid is also considered to present the simulation results. A comparison of the result from the proposed approach with the method based on proportional sharing principle (PSP) [5] is presented. In addition, it is also compared with an approach obtained by incorporation of reactive power loss allocation to the buses based on linear proportional allocation to the circuit based approach presented in Chu et al. [12].

2 Issues in reactive power support and loss allocation

Reactive power support in a deregulated power system can be considered under the separate market called reactive power ancillary service market. For fair and equitable allocation of power, it is necessary to trace the path of power supplied from the source bus to sink bus. Though it is difficult, the power can be traced based on power flow tracing techniques. Based on different approaches, several methods have been proposed for active power tracing, which are utilized for further studies like active power loss allocation, transmission cost allocation, congestion management analysis etc. The main requirement for any tracing/allocation method is that before application of the methodology, the sources and sinks in the system are to be identified. In case of real power, the sources and sinks in the system under consideration are fixed irrespective of the system operating point. The generators are the sources and loads are the sinks. Whereas in case of reactive power, the sources are indefinite like generators, line charging capacitances, synchronous condensers, switchable and variable capacitors, flexible alternating current transmission system (FACTS) devices, and so on [6]. Because of which reactive power sources and flow directions are not certain, and they keep changing continuously depending on the system operating conditions. Hence, identifying the reactive sources for every sink is a big challenge. As reported in Bialek [5], the tracing of reactive power is not the same as the tracing of active power because the amount of reactive power loss in the lines is significant. To handle this issue, many approaches have been proposed in the literature [5–10]. This work mainly focuses on reactive power support and loss allocation rather than tracing the reactive power flow contribution from the sources. In proportional sharing method, the network is assumed to be lossless. Due to which, the amount of reactive power loss taking place in the actual transmission lines is allocated to load buses. One more issue is that unlike active power, the reactive power flow often involves bidirectional flow i.e., reactive power flows into a branch or out of a branch towards both end nodes. Handling of bidirectional flow of reactive power in the method based on proportional sharing principle to trace the reactive power allocation is bit difficult. However, it has been addressed properly while computing the contribution matrix. In the proposed approach based on modified Y-bus matrix, the bidirectional reactive power flow is inherently addressed in the methodology itself.

3 Computation of reactive power support and loss allocation

3.1 Equivalent model of transmission line and system node representation

The line charging capacitances can be treated as sources of providing reactive power to the system. So, while calculating the contribution of reactive sources towards the reactive sink, the line charging capacitances must be considered as reactive sources. The equivalent model of transmission line is shown in Figure 1. The reactive powers ($Q_{c,m}$ and $Q_{c,n}$) produced by the line shunt admittances ($Y_{sh}/2$) are transferred into the nearby nodes with an assumption that the voltages of the shunt admittances are equal to the nearby nodal voltages. The nodal voltages can be obtained by the power flow calculation.

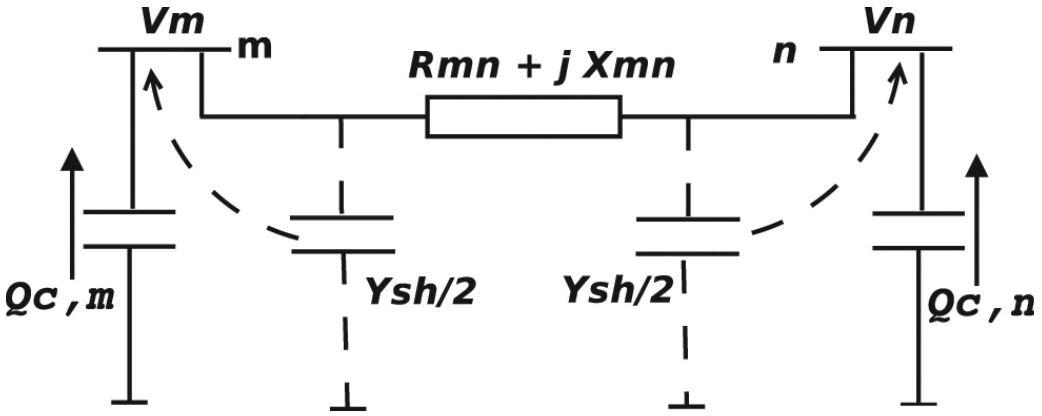


Figure 1: Equivalent model of transmission line.

$$Q_{c,m} = \Im(V_m^2 Y_{sh}/2) \tag{1}$$

$$Q_{c,n} = \Im(V_n^2 Y_{sh}/2) \tag{2}$$

Based on the net reactive power support at the node (considering all the reactive sources, reactive power support produced by the line charging capacitances and reactive sinks), the node can act as either reactive source (that injects reactive power into the system) or reactive sink (that absorbs reactive power from the system). The net reactive power support at the node is defined as,

$$Q_i^{net} = \begin{array}{c} Q_{Gi} \;\; if \;\; (Q_i^{inj} - Q_i^{absorp} + Q_{ci}) < 0 \\ OR \\ Q_{Li} \;\; if \;\; (Q_i^{inj} - Q_i^{absorp} + Q_{ci}) \le 0 \end{array} \tag{3}$$

where Q_i^{inj} is the sum of all reactive power injection at bus *i* by the elements like generators, static capacitors etc.; Q_i^{absorp} is the sum of all reactive power absorption at bus *i* by the elements like shunt reactors, reactive load demand etc.; Q_{ci} is the reactive power support produced by the line charging capacitance at bus *i*; Q_{Gi} is the net injection of the reactive source bus (capacitive in nature) and Q_{Li} is the net absorption of the reactive sink bus (inductive nature).

3.2 Determining the reactive power support from reactive sources and loss allocation to load/sink buses

Consider a system comprising of N number of total buses with NG be the number of generator/source buses, and NL be the remaining load/sink buses. For a given system under steady state operating condition, the network steady state performance equation is given by;

$$[I_G] \;=\; [Y_{GG}][V_G] + [Y_{GL}][V_L] \tag{4}$$

$$[I_L] \;=\; [Y_{LG}][V_G] + [Y_{LL}][V_L] \tag{5}$$

where $[I_G] = [I_1, \dots, I_{NG}]^t$ is the injected currents of generator/source buses; $[I_L] = [I_{NG+1}, \dots, I_N]^t$ is the injected currents of load/sink buses; $[V_G] = [V_1, \dots, V_{NG}]^t$ is the complex generator/source bus voltages; $[V_L] = [V_{NG+1}, \dots, V_N]^t$ is the complex load/sink bus voltages; and $[Y_{GG}]$, $[Y_{GL}]$, $[Y_{LG}]$, $[Y_{LL}]$ are the corresponding partitioned matrices of the bus admittance matrix.

Equation (4) is rewritten in terms of load bus currents and generator bus voltages as

$$[I_G] \;=\; [K_{GL}][I_L] + [Y''_{GG}][V_G] \tag{6}$$

where $[K_{GL}] = [Y_{GL}][Z_{LL}]$, $[Y''_{GG}] = [Y_{GG}]–[Y_{GL}][Z_{LL}][Y_{LG}]$ and $[Z_{LL}] = [Y_{LL}]^{-1}$

The main objective of the proposed work is to get the generators contribution to meet load demand and losses in the system. In order to do so, from circuit theory analysis, the generator bus voltage (V_G) in eq. (6) is being replaced as a function of load buses voltage i.e., $V_G = f(V_L)$. A possible way to deduce generator bus voltage as a function of load buses voltage, is to apply superposition theorem. However, it requires replacing all generators current injection into its equivalent admittances in the circuit. Using readily available load flow results, the equivalent shunt admittance Y_{Gj} of generator node j can be calculated using the following

$$Y_{Gj} = \frac{1}{V_{Gj}}\left(\frac{-S_{Gj}}{V_{Gj}}\right)^{*} \tag{7}$$

where (*) means conjugate, S_{Gj} is the generator apparent power at node j and V_{Gj} is the generator voltage at node j.

Now, these equivalences are added to the corresponding diagonal entries of Y-bus matrix. Then from eq. (4), the generator bus voltage V_G as a function of load buses voltage V_L can be solved. This is given as

$$[V_G] \;=\; -[Y'_{GG}]^{-1}[Y_{GL}][V_L] \tag{8}$$

where $[Y'_{GG}]$ is the modified sub-matrices of $[Y_{GG}]$. From eq. (8), it is assumed that

$$[Y_B] \;=\; -[Y'_{GG}]^{-1}[Y_{GL}] \tag{9}$$

Then, eq. (8) can be written as

$$[V_G] \;=\; [Y_B][V_L] \tag{10}$$

The voltage contribution to the generator bus from each load bus voltages is expanded as,

$$V_{Gj} \;=\; \textstyle\sum_{i=1}^{NL} Y_{Bj,i} * V_{Li} \tag{11}$$

It can be seen from eq. (11) that the original generator voltage at bus j is the sum of individual voltage contribution from all load buses. By substituting eq. (10) into eq. (6), the generator current can be expressed as,

$$I_G \;=\; [K_{GL}][I_L] + [Y_C][V_L] \tag{12}$$

where $[Y_C] = [YGG][Y_B]$

In order to determine the generators share/contribution to meet the load demand and losses, the vectors $[I_L]$ and $[V_L]$ should be considered as a diagonal matrix. Taking a conjugate of eq. (12) and pre-multiplying it by the diagonal generator voltage matrix ($[V_G]$). The generators complex power can be obtained by

$$\begin{aligned} [V_G]_{G\times G}[I_G^*]_{G\times L} &= [S_{gen-contrb}]_{G\times L} \\ &= [V_G]_{G\times G}[K_{GL}^*]_{G\times L}[I_L^*]_{L\times L} + [V_G]_{G\times G}[Y_C^*]_{G\times L}[V_L^*]_{L\times L} \end{aligned} \tag{13}$$

The reactive power contribution of all generators to the load buses can be given as,

$$[Q_{gen-contrb}]_{G\times L} \quad = Im \ \left([S_{gen-contrb}]_{G\times L}\right) \tag{14}$$

With further simplification of eq. (14), the reactive power contribution from generator *j* to load bus *i* is as follows:

$$Q_{gen-contrb_{(j)}} \quad = \quad \sum_{i=1}^{NL} Q_{gen-contrb_{(ji)}} \tag{15}$$

From eq. (15), the reactive power loss allocated to each load bus *i* can be expressed as,

$$Q^{L}_{loss(i)} \quad = \quad \sum_{j=1}^{NG} Q_{gen-contrb_{(ji)}} - Q_{Li} \tag{16}$$

where Q_{Li} is the reactive power load demand at bus *i*.

All the procedures of the proposed approach as explained in the 3 are summarized in flow chart as shown in Figure 2.

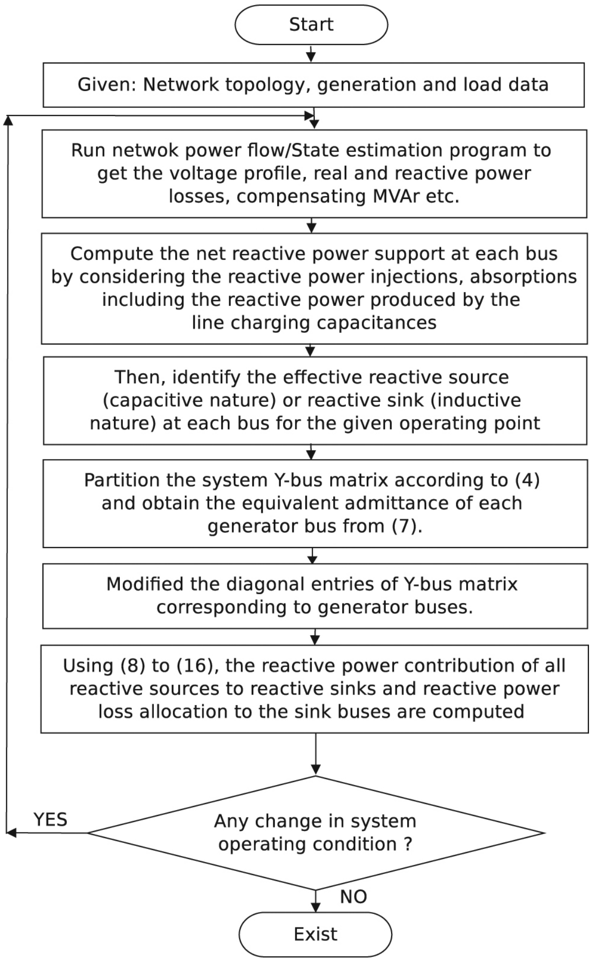


Figure 2: Flow chart for the proposed approach.

4 System studies and discussions

Several IEEE standard systems [17] and Indian practical systems [18] with different voltage levels namely, 400/220/132 kV etc. have been used to verify the effectiveness of the proposed approach. A 11-bus equivalent system, which is a part of Indian southern region power grid (SR 11-bus equivalent system) and a large 259-bus equivalent system of Indian western region power grid (WR 259-bus equivalent system) are considered here to present the effectiveness of the proposed approach. The operating point of the system may be generated by running state estimation/load flow program based on loading parameters or from optimal power flow

(OPF) methods depending on the system operator desired objectives [19], which is not the scope of the paper. We presume that the operating point of the system under consideration is readily available for the studies. The result analysis on the system is carried in two steps. First, determining the reactive power support allocation that reactive sink bus receives from various reactive sources and second, the reactive power loss allocation to load/sink buses. The simulation results are compared with the method based on proportional sharing principle and the circuit based approach [12]. However, it is to be noted that the circuit based approach presented in Chu et al. [12], the system reactive power loss is not considered while allocating the reactive power support to load/sink buses. In order to compare the proposed approach (which takes care of system reactive power loss during the allocation process) with the existing circuit based approach [12], the reactive power loss is allocated to buses based on linear proportional allocation. Since the algebraic summation rule doesn't apply for apparent power [2], the reactive power term is considered while allocating the reactive power loss [12].

4.1 11-bus equivalent system of Indian southern power grid

A 11-bus, 400 kV equivalent system, which is a part of Indian southern power grid is considered to illustrate the proposed approach. The single-line diagram of the system is shown in Figure 3. The system data is taken from Moger [18]. The southern region power grid covers the electrical network of four South Indian States such as Karnataka, Andra Pradesh, Tamil Nadu, and Kerala. The system comprising of four generators and 14 transmission lines (which are of 400 kV lines connecting the Karnataka, Andra Pradesh, Tamil Nadu and Kerala States). The real and reactive loads are connected at seven locations. The shunt reactors are connected at few buses for transient over voltage protection.

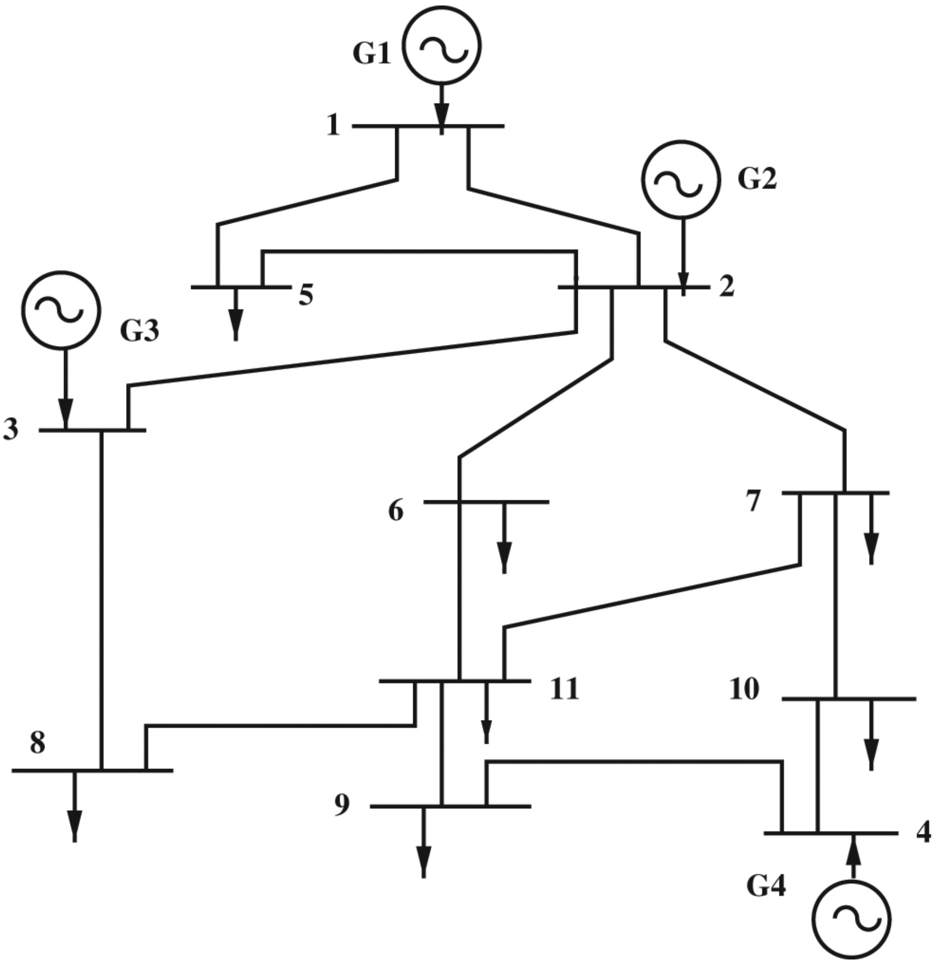


Figure 3: Single-line diagram of 11-bus equivalent system.

The load flow analysis on the system is carried out. Based on the results, the reactive sources and reactive sinks in the system are identified. For discussion purpose, only the load flow result of the system under peak loading condition (LSF=1.7) is shown in Table 1 along with the identification of the reactive sources and sinks. It can be seen from the Table that under peak loading condition, two additional reactive sources (located at buses 6 and 7) are formed in addition to four conventional generator reactive sources. For quick reference and comparison, the reactive power support allocation from the circuit based approach [12] is shown in Table 2 along with separating the other reactive sources. The reactive power loss allocation to the sink/load buses is also shown in Table 2.

Table 1: Load flow result of 11-bus equivalent system with reactive sources and sinks identification for LSF=1.7 (peak load condition).

Bus No.	**Voltage Mag.(p.u.)**	**Angle (deg.)**	**Generation PG (MW)**	**QG (MVAr)**	**Load PD (MW)**	**QD (MVAr)**	**Net MVAr**	**Reactive type**
1	1	0.0	1330.5	57.9	–	–	168.8	Source
2	1	-12.2	535.5	546.5	–	–	750.0	Source
3	1	-13.4	244.8	-51.0	–	–	140.1	Source
4	1	-28.0	795.6	471.9	–	–	613.4	Source
5	0.942	-13.4	–	–	657.9	260.1	-222.2	Sink
6	0.918	-25.3	–	–	91.8	30.6	16.0	Source
7	0.934	-25.5	–	–	183.6	61.2	104.9	Source
8	0.873	-36.3	–	–	275.4	107.1	-5.5	Sink
9	0.927	-33.7	–	–	428.4	137.7	-173.1	Sink
10	0.913	-33.4	–	–	489.6	168.3	-187.5	Sink
11	0.872	-36.3	–	–	688.5	275.4	-198.7	Sink
		Total	2,906.4	1,025.2	2,815.2	1,040.4		
P-loss = 91.206 MW and Q-loss = 1,006.22 MVAr								

Table 2: Reactive power support and loss allocation from the circuit based approach [12] for LSF=1.7 (peak load condition).

(a) Reactive power support allocation

Reactive sink bus	Net demand (MW, MVAr)	Generator reactive source				Other reactive source		Total (MVAr)
		G1	G2	G3	G4	bus 6	bus 7	
5	657.9, 222.2	121.5	100.7	0.0	0.0	0.0	0.0	222.2
8	275.4, 5.5	0.0	0.0	10.7	-7.7	1.5	0.9	5.5
9	428.4, 173.1	0.0	0.0	8.5	141.2	10.5	12.9	173.1
10	489.6, 187.5	0.0	0.0	0.0	104.1	0.0	83.4	187.5
11	688.5, 198.7	0.0	0.0	42.9	37.4	53.1	65.3	198.7
Total	2539.8, 786.9	121.5	100.7	62.1	275.0	65.2	162.5	786.9

(b) Reactive power loss allocation based on linear proportional allocation

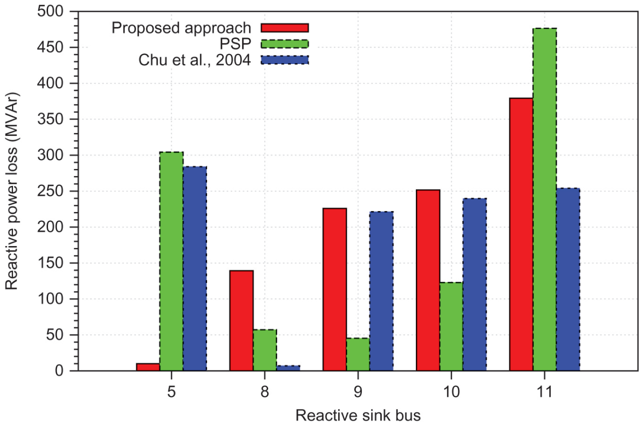
Reactive sink bus	Net demand (MVAr)	Generator reactive source				Other reactive source		Q-loss (MVAr)
		G1	G2	G3	G4	bus 6	bus 7	
5	222.2	155.3	128.8	0.0	0.0	0.0	0.0	284.1
8	5.5	0.0	0.0	13.7	-9.9	2.0	1.2	7.0
9	173.1	0.0	0.0	10.8	180.6	13.4	16.5	221.3
10	187.5	0.0	0.0	0.0	133.2	0.0	106.6	239.8
11	198.7	0.0	0.0	54.8	47.8	67.9	83.5	254.0
Total	786.9	155.3	128.8	79.4	351.6	83.3	207.8	1006.2

The reactive power support received at each load (sink) buses from all generator reactive sources from the proposed approach is shown in Table 3. The allocation of reactive power loss to load/sink buses is also shown in Table 3. The sum of reactive power contributed by each reactive source buses to all reactive sink buses is in agreement with its net reactive power support at the respective reactive source buses. Similarly, the sum of reactive power loss at the sink buses is in agreement with the total reactive power loss calculated by power flow method. As reported in Kirschen and Strbac [6], the generators are sources for real power but may be sources or sinks for reactive power. The partial reactive power support received at bus 5 from the reactive source bus G4 is -72.1 MVAr, so it can be interpreted as generator G4 acts as a sink instead of source for bus 5 for that partial contribution/support. However, the total/overall contribution from all the reactive source buses to sink bus 5 is positive. Since the proposed approach is based on superposition theorem applied to linearized system model, the partial contribution represents the impact (not the share as in case of Bialek [5]) of a particular generator/reactive source to meet the load demand in accordance with circuit characteristics. Comparison is also carried out with the method based on proportional sharing principle and the circuit based approach [12]. The results of the comparison are also presented in Table 3. The comparative analysis of the reactive power loss allocation to load/sink buses from the proposed approach and other existing methods/approaches is shown in Figure 4.

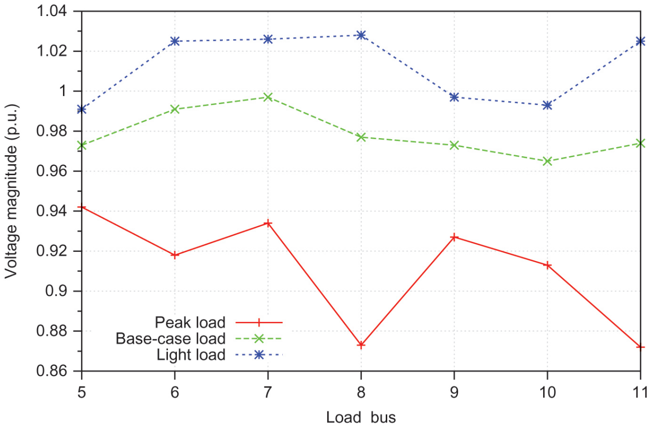
Table 3: Contribution of reactive sources to reactive sinks and allocation of reactive power loss for LSF=1.7 (peak loading condition).

Proposed approach									
Reactive sink bus	Net demand (MVAr)	Generator reactive source				Other reactive source		Total (MVAr)	Q-loss (MVAr)
		G1	G2	G3	G4	bus 6	bus 7		
5	222.2	73.7	155.2	17	-72.1	13	45.4	232.2	10
8	5.5	-11.8	70.6	13.5	58	1.5	13	144.8	139.3
9	173.1	12.9	140.8	27.8	200.5	1.6	15.5	399.1	226
10	187.5	26.3	159.7	31.6	207.3	2.9	11.5	439.2	251.7
11	198.7	67.7	223.7	50.2	219.7	-2.9	19.5	577.9	379.2
Total	786.9	168.8	750	140.1	613.4	16	104.9	1793.1	1006.2
Proportional sharing principle (PSP)									
Reactive sink bus	Net demand (MVAr)	Generator reactive source				Other reactive source		Total (MVAr)	Q-loss (MVAr)
		G1	G2	G3	G4	bus 6	bus 7		
5	222.2	163.9	360.5	2.1	0	0	0	526.5	304.3
8	5.5	0	0	62.8	0	0	0	62.8	57.3
9	173.1	0	0	0	218.5	0	0	218.5	45.4
10	187.5	0.7	58.5	0.3	225.8	0	25	310.4	122.9
11	198.7	4.2	331	74.8	169.1	16	79.9	674.9	476.3
Total	786.9	168.8	750	140.1	613.4	16	104.9	1793.1	1006.2
Circuit based approach [12] plus reactive power loss allocation based on linear proportional allocation									
Reactive sink bus	Net demand (MVAr)	Generator reactive source				Other reactive source		Total (MVAr)	Q-loss (MVAr)
		G1	G2	G3	G4	bus 6	bus 7		
5	222.2	276.8	229.5	0.0	0.0	0.0	0.0	506.3	284.1
8	5.5	0.0	0.0	24.5	-17.7	3.5	2.1	12.5	7.0

9	173.1	0.0	0.0	19.3	321.8	23.9	29.5	394.5	221.4
10	187.5	0.0	0.0	0.0	237.3	0.0	190.0	427.3	239.8
11	198.7	0.0	0.0	97.7	85.1	121.0	148.8	452.7	254.0
Total	786.9	276.8	229.5	141.5	626.6	148.5	370.4	1793.1	1006.2



(a) Reactive power loss allocation for LSF=1.7



(b) Voltage profile of SR 11-bus equivalent system

Figure 4: Reactive power loss allocation for LSF=1.7 and voltage profile of SR 11-bus equivalent system.

Discussions: To meet the power demand at sink bus 9, the reactive power contribution of individual generators is G1= 12.9 MVAr, G2= 140.8 MVAr, G3= 27.8 MVAr, G4= 200.5 MVAr and contribution from other reactive sources is bus 6= 1.6 MVAr and bus 7= 15.5 MVAr. The total partial contribution from all the reactive sources is 399.1 MVAr. The reactive power demand at bus 9 is 173.1 MVAr and rest of the power is lost in the transmission corridors while power is being transferred from source to load due to inductive nature of the transmission lines. Since this work mainly focuses on reactive power analysis, only reactive power contributions are shown and active power contributions are not in the scope of this paper.

In case of sink bus 8, the net reactive power demand at that bus is small i.e., 5.5 MVAr. The total reactive power contribution from all the reactive sources to meet the load demand is 144.8 MVAr and the loss allocated to sink bus 8 is 139.3 MVAr. This shows that reactive power loss also depends on active power flows/demand in the network. Even though the net reactive power demand at the bus is small, there is considerable amount of active power demand at that bus i.e., 275.4 MW. To meet this totally, bus 8 is responsible for 139.3 MVAr reactive power loss in the system. On the contrary, the net reactive power demand at sink bus 5 is 222.2 MVAr. The reactive power loss allocated to the bus from the proposed approach is very less i.e., 10 MVAr, and that from other existing methods/approaches is 304.3 MVAr [5] and 284.1 MVAr [12]. The reason is that the sink bus 5 (Hyderbad-AP) is geographically very close to two generator reactive source buses G1 (Ramagundam-AP) and G2 (Nagarjunasagar-AP). As a result, bus 5 gets maximum share from these two generator reactive source buses (G1 and G2) and also gets some share from other reactive sources in the network in accordance with the circuit characteristics. Obviously, bus 5 may not contribute much to the losses in the lines to meet its demand. Even though from other existing methods/approaches only G1 and G2 contribute maximum to meet the demand, the reactive power loss allocated to sink bus 5 from these methods/approaches is pretty high (i.e., 304.3 MVAr loss from Bialek [5] and 284.1 MVAr loss from Chu et al. [12]). This is because during the allocation process, the other existing methods/approaches do not consider the circuit characteristics behavior and allocations are carried out based on specific assumptions. Similar observations can also be seen from Table 3 for other sink buses.

Case studies are generated for the system under light load condition to peak load condition with the help of load scale factor (LSF). The voltage profile of the system under different loading conditions is shown in Figure 4. The reactive power support and loss allocation along with the comparison of results for the system under different loading conditions are given in Table 4 and Table 5. The comparative analysis of the reactive power loss allocation to load/sink buses for the system under base-case and light load conditions is also shown in Figure 5.

Table 4: Contribution of reactive sources to reactive sinks and allocation of reactive power loss for LSF=1.0 (base-case load condition).

Proposed approach

Reactive sink bus	Net demand (MW, MVAr)	Generator reactive source				Other reactive source			Total (MVAr)	Q-loss (MVAr)
		G1	G2	G3	G4	bus 6	bus 7	bus 8		
5	387, 112.56	4.80	34.95	-1.25	-7.05	13.90	51.02	36.66	133.04	20.48
9	252, 120.01	22.77	43.15	9.43	80.96	5.51	27.31	5.25	194.38	74.36
10	288, 120.47	20.75	46.09	8.94	80.67	7.25	29.36	11.00	204.06	83.58
11	405, 66.25	-2.90	46.58	7.61	63.64	9.64	45.30	11.38	181.26	115.00
Total	1,332, 419.30	45.43	170.77	24.73	218.21	36.30	152.99	64.30	712.72	293.43

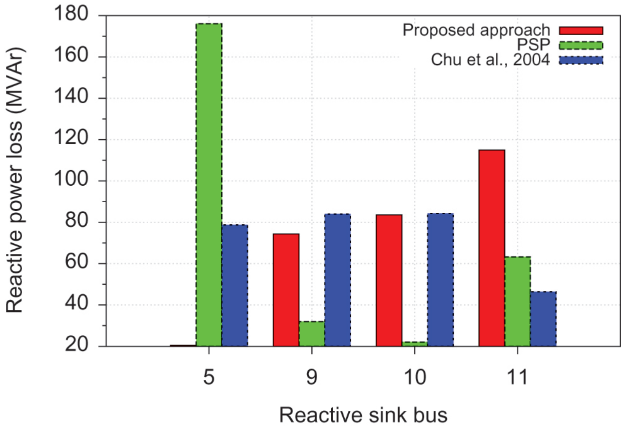
Proportional sharing principle (PSP)

Reactive sink bus	Net demand (MW, MVAr)	Generator reactive source				Other reactive source			Total (MVAr)	Q-loss (MVAr)
		G1	G2	G3	G4	bus 6	bus 7	bus 8		
5	387, 112.56	45.43	166.09	2.67	0.00	16.88	56.22	1.33	288.61	176.05
9	252, 120.01	0.00	0.59	2.80	133.12	2.47	4.99	8.00	151.99	31.98
10	288, 120.47	0.00	0.00	0.00	85.08	0.00	57.49	0.00	142.58	22.10
11	405, 66.25	0.00	4.09	19.26	0.00	16.95	34.29	54.97	129.55	63.30
Total	1,332, 419.30	45.43	170.77	24.73	218.21	36.30	152.99	64.30	712.72	293.43

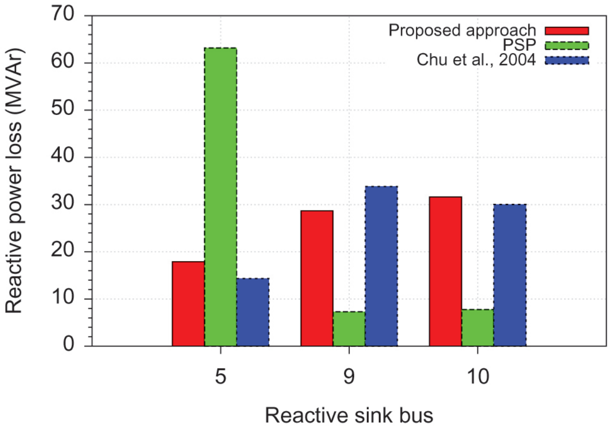
Circuit based approach [12] plus reactive power loss allocation based on linear proportional allocation

Reactive sink bus	Net demand (MW, MVAr)	Generator reactive source				Other reactive source			Total (MVAr)	Q-loss (MVAr)
		G1	G2	G3	G4	bus 6	bus 7	bus 8		
5	387, 112.56	94.56	96.77	0.00	0.00	0.00	0.00	0.00	191.33	78.77
9	252, 120.01	0.00	0.00	0.00	148.60	3.88	4.82	46.69	204.00	83.99
10	288, 120.47	0.00	0.00	0.00	114.68	0.00	90.10	0.00	204.78	84.31
11	405, 66.25	0.00	0.00	0.00	7.61	9.20	11.20	84.61	112.62	46.36
Total	1,332, 419.30	94.56	96.77	0.00	270.90	13.08	106.11	131.30	712.72	293.43

As reported in Bialek [5], the distribution factor generated from the proportional sharing principle is based on topological analysis of the network flow. It represents just the share of a particular generation in the load demand. This factor is always positive. In case of real power, the generators always inject real power to the system and the real power contribution from those sources is positive. However, in case of reactive power, the generator can inject (considered as positive) or absorb (considered as negative) reactive power, which depends on the system operating condition. Even though the topological distribution factors are positive, because of absorbing nature of generators during light load condition, the contribution from those sources is negative. It can be seen from the Table 5 that under light load condition (LSF=0.5), the contribution from the generator reactive sources i.e., G1= −0.35 MVAr, G2= −94.58 MVAr, G3= −24.21 MVAr is negative and the contribution from other reactive sources is positive. It can further be noted that when the system is operating under light load conditions, the amount of reactive power produced by the line charging capacitances is significant. As a result, four additional reactive sources (located at buses 6, 7, 8 and 11) are formed in comparison with two additional reactive sources (located at buses 6 and 7) under peak load condition and three additional reactive sources (located at buses 6, 7 and 8) under base-case load condition.



(a) Reactive power loss allocation for LSF=1.0



(b) Reactive power loss allocation for LSF=0.5

Figure 5: Reactive power loss allocation of SR 11-bus equivalent system.

Inference: After analyzing the system for various case studies from the proposed approach and other existing method/approach, it can be inferred that the reactive power support and loss allocation from the proposed approach is carried out in a systematic manner without any assumptions unlike the methods/approaches in the literature [5, 12]. By intuition, as loading on the system increases, the power loss taking place in the transmission system must increase. The loss allocation to the load/sink buses from the proposed approach is increasing as the system is moving from light load condition to peak load condition in the same severity order of the buses i.e., bus 11, 10, 9 etc. Hence, the proposed approach is very efficient in giving proper signal to the market participants and system operator regarding the system reactive power issues. Therefore, the amount of reactive power loss allocated at load buses can be considered as an indicator to the reactive power deficit/surplus at those buses. Hence, it can be used for identification of weak buses in the system [20].

It can be observed from the results that the two existing approaches [5, 12] are inconsistent particularly with respect to the allocation of reactive power loss i.e., the nodes to which this loss is allocated.

Table 5: Contribution of reactive sources to reactive sinks and allocation of reactive power loss for LSF=0.5 (light load condition).

Proposed approach

Reactive sink bus	Net demand (MW, MVAr)	Generator reactive source				Other reactive source				Total (MVAr)	Q-loss (MVAr)
		G1	G2	G3	G4	bus 6	bus 7	bus 8	bus 11		
5	193.5, 34.53	-35.34	-50.94	-17.03	-28.88	21.44	74.88	51.89	36.41	52.43	17.90
9	126, 81.46	23.53	-17.98	-2.00	28.40	12.55	50.62	25.25	-10.20	110.16	28.70
10	144, 72.22	11.46	-25.66	-5.19	20.17	15.10	56.86	32.38	-1.28	103.85	31.62
Total	463.5, 188.21	-0.35	-94.58	-24.21	19.69	49.09	182.35	109.52	24.92	266.43	78.22

Proportional sharing principle (PSP)

Reactive sink bus	Net demand (MW, MVAr)	Generator reactive source				Other reactive source				Total (MVAr)	Q-loss (MVAr)
		G1	G2	G3	G4	bus 6	bus 7	bus 8	bus 11		
5	193.5, 34.53	-0.35	-94.58	-24.21	0.00	49.09	118.47	45.59	3.68	97.69	63.17
9	126, 81.46	0.00	0.00	0.00	6.41	0.00	0.00	61.81	20.54	88.76	7.30
10	144, 72.22	0.00	0.00	0.00	13.28	0.00	63.88	2.12	0.70	79.98	7.75
Total	463.5, 188.21	-0.35	-94.58	-24.21	19.69	49.09	182.35	109.52	24.92	266.43	78.22

Circuit based approach [12] plus reactive power loss allocation based on linear proportional allocation

Reactive sink bus	Net demand (MW, MVAr)	Generator reactive source				Other reactive source				Total (MVAr)	Q-loss (MVAr)
		G1	G2	G3	G4	bus 6	bus 7	bus 8	bus 11		
5	193.5, 34.53	23.19	25.69	0	0.00	0	0.00	0	0.00	48.87	14.35
9	126, 81.46	0.00	0.00	0	77.92	0	0.00	0	37.40	115.31	33.85
10	144, 72.22	0.00	0.00	0	57.14	0	45.10	0	0.00	102.24	30.02
Total	463.5, 188.21	23.19	25.69	0	135.05	0	45.10	0	37.40	266.43	78.22

The possible reason may be the allocation is carried out with specific assumptions. In case of circuit based approach presented in Chu et al. [12], if the reactive power demand at the load bus is increased, a higher reactive power loss is allocated to that bus and vice-versa. It is based on load demand level rather than their relative location in the network and the network topology is not taken into consideration. This type of method/approach is easy to understand and implement. In case of the method based on proportional sharing principle, the values just represent the share and not the impact of the sources. The assumptions are made while determining the sharing from the generators. Further, the method does not take into consideration of the network performance equations that describe the characteristics of the actual system. However, the validity of the proportional sharing method can be neither proved nor disproved. Even though the method may be efficient for real power allocation/tracing, for reactive power allocation the network characteristics play an important role because of dominant nature of reactance of the transmission lines.

4.2 259-bus equivalent system of Indian western region power grid

A 259-bus, 400/220/132 kV equivalent system of Indian western region power grid is considered to evaluate the reactive power support and loss allocation from the proposed approach. The geographical map of Indian western region power grid is shown in Figure 6. The system data is taken from Moger [18]. The western region power grid covers the electrical network of Indian States such as Maharashtra, Gujarat, Madhya Pradesh, Chhattisgarh, and Goa. The system comprising of 49 generators, which come under central sector generating companies such as NTPC Ltd. (National Thermal Power Corporation Limited), NPCI Ltd. (Nuclear Power Corporation of India Limited), state sector generating companies and joint venture companies as well as independent power producers (IPPs), and 475 transmission lines including transformers, which are of various

(400/220/132 kV) voltage levels. The shunt reactors are connected at few buses for transient over voltage protection. The peak load on the system is considered for the analysis. The load flow summary of the system under peak load condition is shown in Table 6.

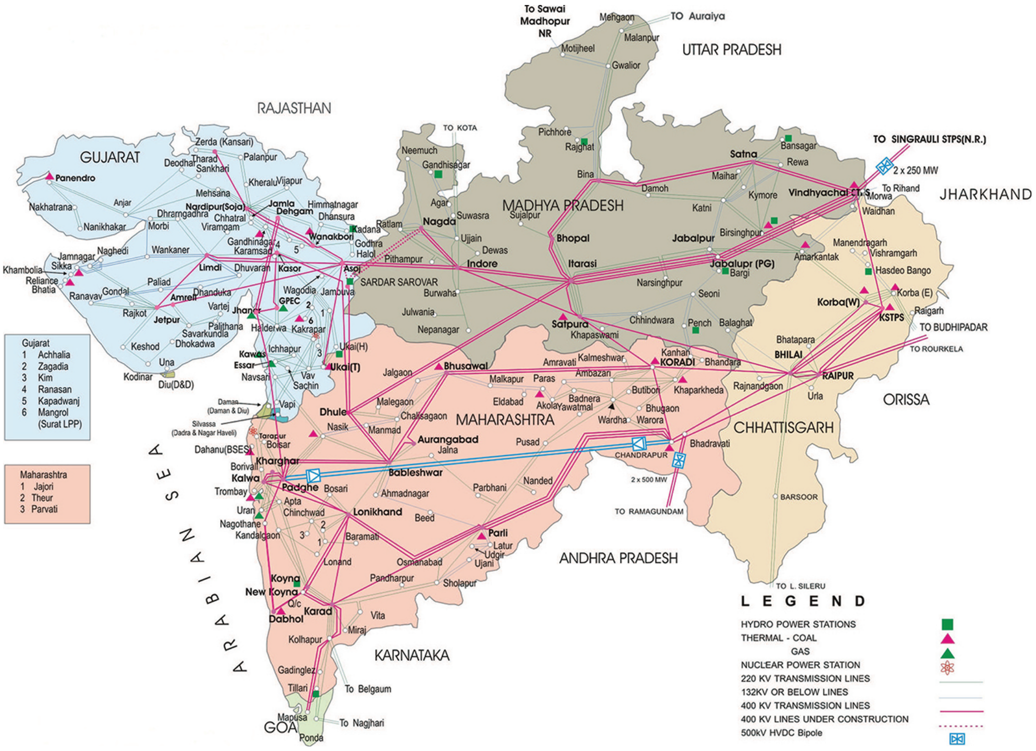


Figure 6: Geographical map of 259-bus equivalent system of Indian western region power grid [21].

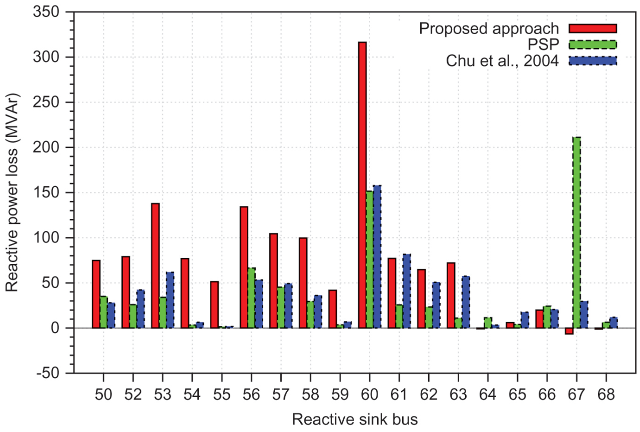
Table 6: System summary of 259-bus practical system of Indian western region power grid.

Load flow summary	
No. of generators:	49
No. of transmission links :	373
No. of Transformers :	102
No. of shunt reactors:	16
No. of shunt capacitors:	11
Total generation:	12568.8 MW and 3977.2 MVAr
Total P-Q load:	12056.1 MW and 5227.9 MVAr
Total power losses:	512.7 MW and 4946.6 MVAr
Load bus voltage, V_{min}:	0.808 (p.u.)
After source/sink conversion	
No. of reactive sources: (including generator buses)	149
No. of reactive sinks:	110
Total reactive power generation:	7857.5 MVAr
Total reactive power demand:	2910.9 MVAr

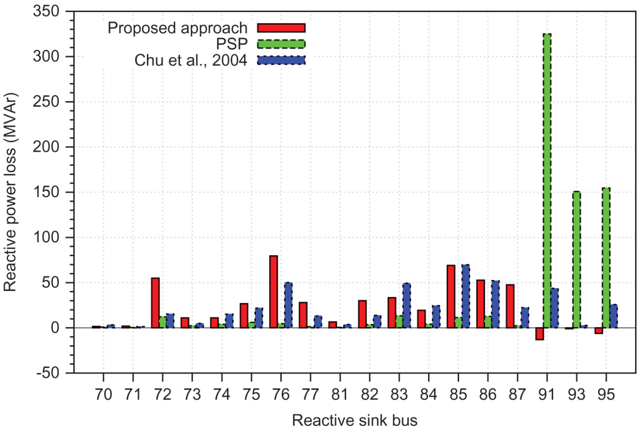
Similar to SR 11-bus equivalent system, the load flow analysis on the system is carried out. Based on the results the reactive sources and reactive sinks in the system are identified. The brief summary of the system after reactive sources and sinks conversion is also given in Table 6. Due to space constraints, only the reactive power loss allocation to the sink buses is discussed. The reactive power loss allocation from the proposed approach for the few important sink buses is shown in Table 7. Comparisons with the method based on proportional sharing principle and the circuit based approach [12] are also shown in Table 7. The reactive power loss allocation to all sink buses is shown in Figure 7, 8 and 9. As it can be observed from the Figures that few sink buses are allocated negative loss. However, the sum of reactive power loss at all sink buses from the proposed approach and other existing methods/approaches is in agreement with the total reactive power loss obtained from load flow result. Similar to marginal loss allocation method [4], the negative loss allocation to buses from the circuit based methods is common in the literature. As described in Conejo et al. [11], the buses are in strategically well positioned location in the system/network are with negative loss, which might be interpreted as cross subsidies. It means these buses should receive monetary incentives for their negative loss because of their impact on reducing the overall network loss. It can be seen from the Table and Figures that if the power demand at load/sink bus is more (including active power), more reactive power loss is allocated to that bus from the proposed approach. Further, it can be seen from the Table 7 that for the power demand at the buses i.e., bus-135=373 MW+j184.42 MVAr, bus-60=246 MW+j92.83 MVAr and bus-182=259 MW+j108.85 MVAr, the corresponding reactive power loss allocated to those respective buses are 318.46 MVAr, 316.46 MVAr, and 217.27 MVAr. Therefore, it can be inferred that the proposed approach also takes into consideration the active power demand and the relative location of the nodes in the network. But in the method based on PSP, more loss is allocated to those load/sink buses which may not have appreciable amount of power demand (including active power). Based on PSP, the sink/load buses, which have more power demand are allocated less reactive power loss. In the circuit based approach [12], if the reactive power demand at load/sink bus is more, more reactive power loss is allocated to that bus. Hence, we can come to the conclusion that even in such a large practical system, the allocation from the proposed approach is consistent as compared with other existing methods/approaches in the literature.

Table 7: Reactive power loss allocation to sink buses for WR 259-bus equivalent system under peak loading condition (few important buses).

Reactive sink bus	Net demand (MW, MVAr)	Proposed approach		PSP		Approach [12]	
		Total (MVAr)	Q-Loss (MVAr)	Total (MVAr)	Q-Loss (MVAr)	Total (MVAr)	Q-Loss (MVAr)
60	246, 92.83	409.22	316.40	244.34	151.51	250.57	157.74
96	85.5, 13.22	42.12	28.90	181.46	168.24	35.69	22.47
98	63, 47.83	73.74	25.90	217.06	169.23	129.12	81.28
103	126, 44.41	117.88	73.47	90.86	46.44	119.89	75.47
113	144, 53.38	127.61	74.23	170.31	116.92	144.09	90.71
129	185, 104.21	266.15	161.94	134.08	29.87	281.30	177.09
135	373, 184.42	502.89	318.46	229.64	45.22	497.81	313.39
138	112, 57.66	139.64	81.98	75.84	18.19	155.64	97.98
139	194.4, 79.14	175.07	95.94	340.51	261.38	213.62	134.48
182	259, 108.85	326.12	217.27	202.97	94.12	293.81	184.97
183	141, 68.27	182.73	114.45	98.82	30.55	184.28	116.01
201	72, 10.16	30.56	20.41	106.91	96.75	27.42	17.26

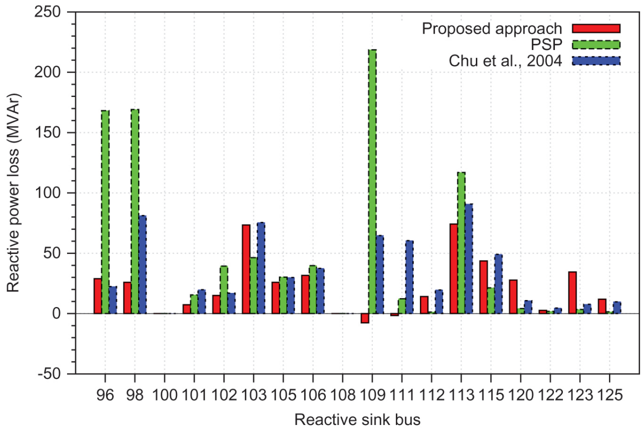


(a) Reactive power loss allocation for buses 50-68

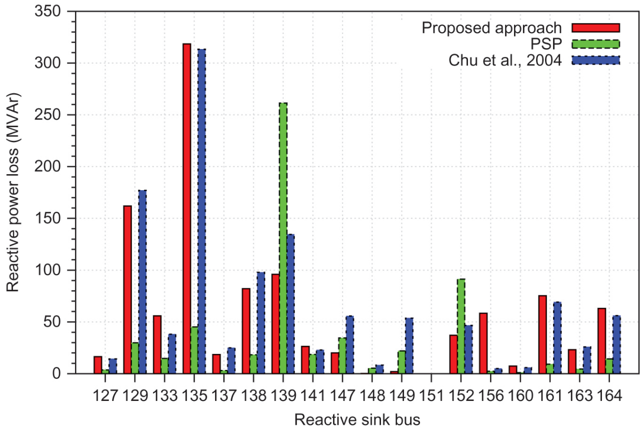


(b) Reactive power loss allocation for buses 70-95

Figure 7: Reactive power loss allocation of WR 259-bus equivalent system.

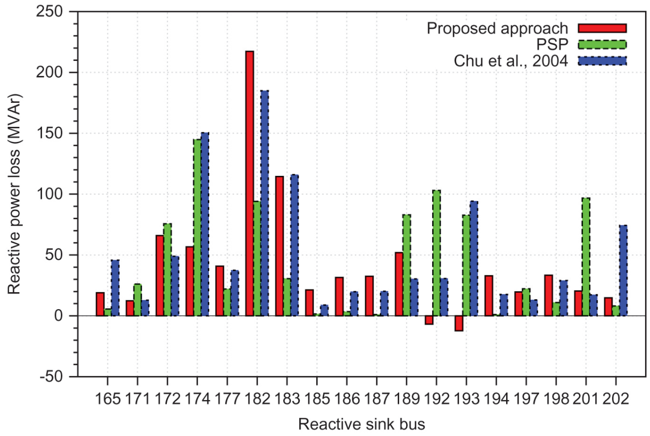


(a) Reactive power loss allocation for buses 96-125

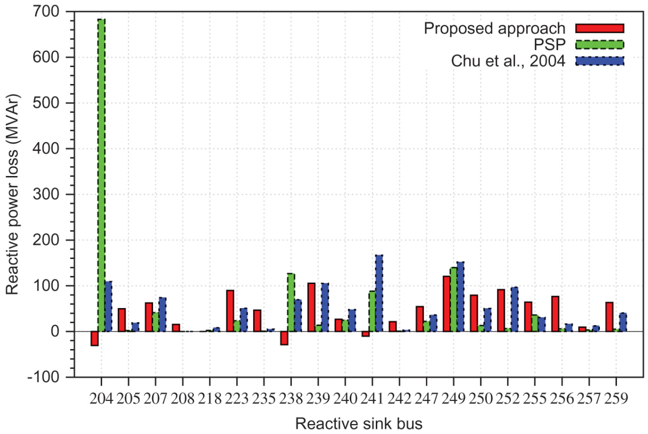


(b) Reactive power loss allocation for buses 127-164

Figure 8: Reactive power loss allocation of WR 259-bus equivalent system.



(a) Reactive power loss allocation for buses 165-202



(b) Reactive power loss allocation for buses 204-259

Figure 9: Reactive power loss allocation of WR 259-bus equivalent system.

5 Conclusions

In this paper, a new approach has been proposed using modified Y-bus matrix to compute the reactive power contribution from various reactive sources to meet the reactive load demand and losses. Further, the allocation of reactive power loss to load/sink buses is also computed. The proposed approach is based on solved load flow, and all its computations are based on the circuit theory and equivalent system admittance matrix without any assumptions. The advantage of the proposed approach is that it takes into consideration the network inherent characteristics in terms of network equations directly. The reactive power support allocation from the proposed approach may be utilized for detailed pricing of reactive power ancillary services in a deregulated electricity market in a more transparent manner. In addition, the reactive power loss allocated at the load/sink buses can also be used for identification of weak buses in the system. Detailed case studies are carried out on various systems under different system operating conditions. Comparisons with other existing methods/approaches in the literature show that the proposed approach is fair and accurate.

References

1. FERC. Principles for efficient and reliable reactive power supply and consumption. USA: 2005 FERC Staff ReportsDocket No. AD05-1-000.
2. Elgerd Olle Ingemar. Electic energy systems theory-an introduction. New York: McGraw-Hill Book Company, 1982 .
3. Ilic MD, Galiana F, Fink L. Power systems restructuring: engineering and economics. 1998;448(Springer).
4. Galiana FD, Conejo AJ, Kockar I. Incremental transmission loss allocation under pool dispatch. IEEE Trans Power Syst. 2002;17(1):26–33.
5. Bialek J. Tracing the flow of electricity. IEE Proce Gener Trans Distrib. 1996;143(4):313–20.
6. Kirschen D, Strbac G. Tracing active and reactive power between generators and loads using real and imaginary currents. IEEE Trans Power Syst. 1999;14(4):1312–1319.

7. Wu FF, Ni Y, Wei P. Power transfer allocation for open access using graph theory-fundamentals and applications in systems without loopflow. IEEE Trans Power Syst. 2000;15(3):923–9.
8. Gubina F, Grgic D, Banic I. A method for determining the generators' share in a consumer load. IEEE Trans Power Syst. 2000;15(4):1376–81.
9. Panto's M, Verbic G, Gubina F. Modified topological generation and load distribution factors. IEEE Trans Power Syst. 2005;20(4):1998–2005.
10. Abdelkader SM. Transmission loss allocation through complex power flow tracing. IEEE Trans Power Syst. 2007;22(4):2240–8.
11. Conejo AJ, Galiana FD, Kockar I. Z-bus loss allocation. IEEE Trans Power Syst. 2001;16(1):105–10.
12. Chu WC, Chen BK, Liao CH. Allocating the costs of reactive power purchased in an ancillary service market by modified y-bus matrix method. IEEE Trans Power Syst. 2004;19(1):174–9.
13. De M, Goswami S. Reactive support allocation using improved y-bus matrix method. IET Gener Trans Distrib. 2011;5(4):448–60.
14. Thukaram D, Vyjayanthi C. Relative electrical distance concept for evaluation of network reactive power and loss contributions in a deregulated system. IET Gener Trans Distrib. 2009;3(11):1000–19.
15. Lo K, Alturki Y. Towards reactive power markets. Part 1: reactive power allocation. IEE Proc Gener Trans Distrib. 2006;153(1):59–70.
16. Moger T, Dhadbanjan T. An improved approach for evaluation of reactive power sources contribution to reactive load and loss. 2013 Annual IEEE India Conference (INDICON). 2013;1–6.
17. Electrical Engineering Department, University of Washington. Power systems test case archive. Available at: http://www.ee.washington.edu/research/pstca.
18. Moger T. Reactive power planning and operation of power systems with wind farms for voltage stability improvement (Ph.D. dissertation). Bangalore: Indian Institute of Science, Department of Electrical Engineering, 2016.
19. Zhang W, Li F, Tolbert LM. Review of reactive power planning: objectives, constraints, and algorithms. IEEE Trans Power Syst. 2007;22(4):2177–86.
20. Moger T, Dhadbanjan T. A novel index for identification of weak nodes for reactive compensation to improve voltage stability. IET Gener Trans Distrib. 2015;9(14):1826–34.
21. Ravikumar B, Thukaram D, Khincha H. Comparison of multiclass SVM classification methods to use in a supportive system for distance relay coordination. IEEE Trans Power Delivery. 2010;25(3):1296–305.