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Field evaluation of unsaturated hydraulic conductivity models and parameter estimation from retention data

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Abstract

Predictions of two popular closed-form models for unsaturated hydraulic conductivity (K) are compared with in situ measurements made in a sandy loam field soil. Whereas the Van Genuchten model estimates were very close to field measured values, the Brooks–Corey model predictions were higher by about one order of magnitude in the wetter range. Estimation of parameters of the Van Genuchten soil moisture characteristic (SMC) equation, however, involves the use of non-linear regression techniques. The Brooks–Corey SMC equation has the advantage of being amenable to application of linear regression techniques for estimation of its parameters from retention data. A conversion technique, whereby known Brooks–Corey model parameters may be converted into Van Genuchten model parameters, is formulated. The proposed conversion algorithm may be used to obtain the parameters of the preferred Van Genuchten model from in situ retention data, without the use of non-linear regression techniques.

1. Introduction

Mathematical models of hydrologic and agricultural systems require knowledge of the relationship of soil unsaturated hydraulic conductivity to volumetric soil moisture content (θ) or soil matric potential (h). Hence, a sustained research effort towards the parameterisation of K has resulted in the development of several laboratory, field and theoretical methods.

Laboratory techniques for determining K (or diffusivity), involve setting up either steady- or transient-state flow systems in which field extracted samples are tested. However, it is now recognised that laboratory tests cannot fully duplicate field conditions. Among field methods, the profile internal drainage test (Rose et al., 1965; Hillel et al., 1972) has come to be accepted as a standard in situ procedure,

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notwithstanding certain inherent limitations. However, routine use of field methods is often inhibited by cost considerations, especially when large areas have to be characterised for their hydraulic properties.

This has led to the development of theoretical methods for estimation of K from basic soil properties, which may be either physical and chemical properties (e.g. percentage of sand, percentage of clay, percentage of organic matter, etc.) or the soil moisture characteristic (SMC). In particular, closed-form equations for K (Brooks and Corey, 1964; Campbell, 1974; Van Genuchten, 1980) in terms of certain descriptive parameters of the SMC have become extremely popular because of their simple form. Such equations have been derived from the statistical pore interaction models of Burdine (1953) and Mualem (1976).

The models of Brooks and Corey (1964) and Van Genuchten (1980) are most suitable, provided a reliable in situ SMC is available. A few studies have compared laboratory or in situ determined K functions with the predictions of the Brooks–Corey model (e.g. Brust et al., 1968; Bruce, 1972; Talsma, 1985; Alexander and Skaggs, 1987). A finding common to all these studies is that the Brooks–Corey model is a fairly accurate predictor of K for coarse-textured soils. However, the performance of the Van Genuchten model in predicting field measured K has not been so widely tested (e.g. Dane, 1980). Further, comparisons between the performances of these two models in field conditions appear to be lacking. It is therefore the intention of this study to test these models with respect to the K function obtained from field measurements, and also to develop a method for estimating model parameters from soil-water retention data.

2. Closed-form models for K

2.1. Van Genuchten model

Van Genuchten (1980) obtained the following closed-form expression for K:

$$K(S_w) = K_s(S_w)^{\rho}[1 - (1 - S_w^{1/m})^m]^2 \tag{1}$$

where $S_w = (\theta - \theta_r)/(\theta_s - \theta_r)$, with subscripts r and s representing residual and saturation values of the soil moisture content θ (cm^3 cm^{-3}), K_s is the hydraulic conductivity (mm day^{-1}) at saturation, ρ is a pore interaction factor, assumed by Van Genuchten to be equal to 0.5, and m is defined by

$$S_w = [1 + (\alpha h)^n]^{-m} \tag{2}$$

where α is a parameter, $n = 1/(1 - \mathrm{m})$, and h is the soil matric potential (kPa).

2.2. Brooks–Corey model

The Brooks–Corey (1964) model for K is

$$K(S_w) = K_s(S_w)^{2/\tau+3} \tag{3}$$

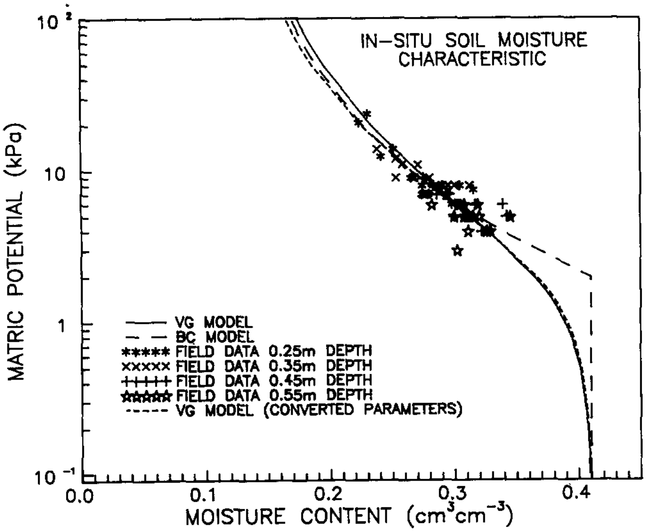

Fig. 1. Field retention data and fitted analytic models for the soil moisture characteristic.

Eq. (3) (which is equivalent to the Campbell (1974) and Laliberte et al. (1966) equations) was obtained by coupling the SMC in the form of Eq. (4) with the Burdine statistical pore interaction model.

$$S_w = \left(\frac{h}{h_e}\right)^{-\tau}, \qquad h > h_e \tag{4a}$$

$$S_w = 1.0, \qquad h \leqslant h_e \tag{4b}$$

where h_e is the air-entry matric potential and τ is a constant.

3. Methods

As part of a water balance study of irrigated areas, components of the field water regime were measured within a farmer's plot, located at Adde Viswanathapura, north of Bangalore City, South India. The soil is a sandy loam, with mean percentages of sand, silt and clay of 54, 26 and 20, respectively, mean bulk density 1.58 g cm^{-3} and mean percentage of organic matter 0.85.

3.1. In-situ SMC

An Institute of Hydrology neutron moisture probe (Didcot Instruments, Abingdon, UK) was used to measure soil moisture in the crop root zone. Around one of the brass access tubes (44.45 mm o.d., 0.9 m length) installed for the purpose, tensiometers (19 mm o.d.), fitted with vacuum pressure gauges, were installed at 0.25, 0.35, 0.45 and 0.55 m depths. These depths also correspond to neutron probe reading positions. Both probe and tensiometer readings were taken at weekly

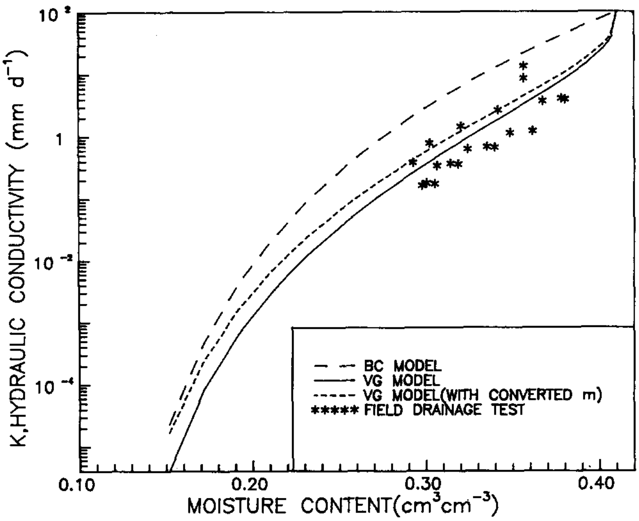

Fig. 2. In situ hydraulic conductivity compared with predictions of closed-form models.

intervals (or, at times, longer intervals) for the period from 5 August to 13 December 1991. During this period a finger millet crop was grown on the plot and the profile was frequently wetted owing to rainfall and irrigation applications. Tensiometer readings expressed in kilopascals were plotted against the corresponding volumetric moisture (Fig. 1). As no clear depth dependence is discernible, it is assumed that a single SMC could represent the entire 0.2–0.6 m soil layer without loss of much accuracy. The above data (Fig. 1), composited for all the measurement depths, were fitted to the analytic SMC models of Brooks and Corey (Eq. (4)) and Van Genuchten (Eq. (2)).

3.2. In situ unsaturated hydraulic conductivity

A profile internal drainage test was conducted in a fallow, levelled (3 m × 3 m) portion of the experimental plot following the procedure laid out by Hillel et al. (1972). Depth-wise soil moisture contents and matric potentials were measured using the neutron probe and tensiometers, at increasing time intervals after the profile had been initially wetted to saturation, for a total drainage period of 20 days. Using these data, K as a function of soil moisture content was calculated and is shown plotted in Fig. 2. After the profile had been initially wetted to saturation, water was ponded on the plot and the infiltration rate measured. This final infiltration rate of 100 mm day^{-1} was assigned to the saturated hydraulic conductivity (K_s).

4. Results and discussion

The saturated moisture content (θ_s) in both the SMC models (Eqs. (2) and (4)), was assumed to equal the average soil porosity (0.41 cm^3 cm^{-3}), as calculated from

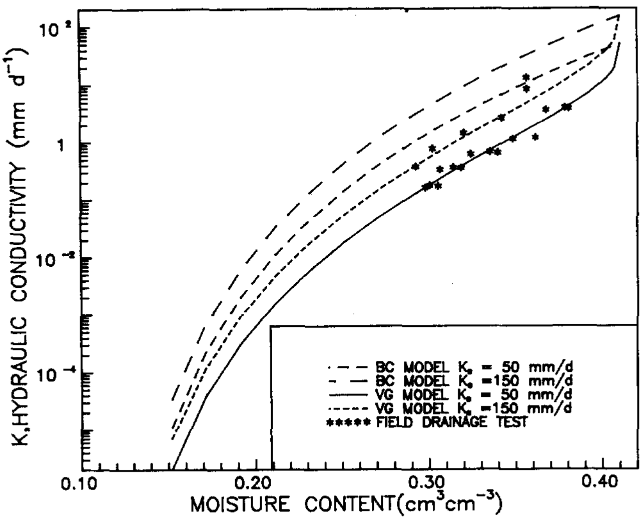

Fig. 3. Predictions of closed-form models with various values of saturated hydraulic conductivity (K_s).

measured values of particle density and bulk density of several soil samples. Following the suggestion of Van Genuchten (1980), θ_r was assumed to equal 0.11 cm^3 cm^{-3}, the equilibrium moisture content at 1500 kPa matric potential as determined from laboratory tests on field soil samples using a pressure plate apparatus.

A non-linear optimisation algorithm was used to estimate the remaining parameters of Eqs. (2) and (4) for the field retention data shown in Fig. 1. The optimal parameter values obtained were: $\alpha = 0.4156$ kPa^{-1}, $n = 1.415$ for Eq. (2); $h_e =$ 2.066 kPa, $\tau = 0.419$ for Eq. (4). Fig. 1 also shows a comparison between the predictions of Eqs. (2) and (4) when used with the optimised parameter set.

4.1. Evaluation of K models

A comparison of K values generated from Eqs. (1) and (3) and the results of the internal drainage field test is shown in Fig. 2. The Van Genuchten (VG) model gives estimates of K that are very close to the mean line through the measured points. The Brooks–Corey (BC) model, on the other hand, overpredicts the in situ K by about one order of magnitude. As the internal drainage test yields K data in the wet range only, model validation for the entire moisture range is not possible. The discrepancy between the VG and BC models decreases in the dry range. This is similar to the convergence of the two underlying SMC models at low moisture contents.

The sensitivity of the results to errors in the value of K_s was tested by setting K_s to values of 50 mm day^{-1} and 150 mm day^{-1} while retaining all other parameters at their earlier values. The results are shown in Fig. 3, which suggests that the performance of the BC model would have improved with a much lower K_s. Pursuing this, an optimisation exercise was undertaken which gave the values of K_s for best fit with the experimental data as 79.94 mm day^{-1} for the VG model and

14.35 mm day^{-1} for the BC model. Though the measured value of K_s is subject to experimental error, the true value is unlikely to be as low as that required by the BC model.

4.2. Evaluation of SMC models

Several studies (e.g. Van Genuchten, 1980; Russo, 1988) have shown that, on average, the VG model is a better descriptor of the laboratory and in situ measured SMCs than the BC model. Additionally, the VG model is a continuous and smooth function, whereas the BC model contains a discontinuity in slope at the air-entry matric potential value (Fig. 1). This discontinuity is an undesirable feature, especially when incorporated into numerical models of flow in the unsaturated zone. However, the most attractive feature of the BC model is its simple form (power equation), which can be linearised by taking logarithms on both sides of the equation, and its parameters estimated by application of simple linear least-squares (LS) regression techniques, though certain statistical problems, as elaborated by Milly (1987), may be encountered in fitting the BC model to retention data using LS techniques. As the VG equation cannot be linearised by simple transformations, parameter estimation for large data sets involves use of complex non-linear LS regression techniques.

In this study, a simple method for obtaining the VG model parameters is described, which does not require the use of non-linear LS techniques. The method involves initially fitting the BC model to the experimental retention data and then conversion of the BC model parameters to equivalent VG model parameters. Van Genuchten (1980), in presenting his model for the SMC, also briefly commented on its equivalence with the BC model. More recently, Lenhard et al. (1989) have elaborated on this point and presented empirical equations for conversion between VG and BC model parameters. However, their equations are more suitable for converting given VG parameters into equivalent BC parameters and therefore cannot be used directly in the present context.

4.3. Proposed conversion algorithm

The present approach is essentially an adaptation of the semi-graphical parameter estimation procedure described by Van Genuchten (1980). In this procedure, equations have been developed for the VG model parameters in terms of the slope of the saturation vs. logarithm of matric suction plot. Van Genuchten suggested graphical determination of this slope (S_p) at a point halfway between θ_s and θ_r (i.e. at $S_w = 0.5$), from an experimental retention curve. As Van Genuchten did not elaborate on how such a retention curve is obtained with reference to in situ obtained neutron probe–tensiometer data exhibiting scatter, we suggest that the SMC described by the BC model be used in its place. It is assumed that independent estimates of θ_s and θ_r are available.

First, the BC model (Eq. (4a)) is to be fitted to the given experimental retention data and model parameters h_e and γ determined by long-linear LS regression analysis. The

Table 1
BC model parameters and calculated equivalent VG model parameters for four soils given by Brooks and Corey (1964)

Soil	BC model		VG model	
	h_e (kPa)	τ	α (kPa^{-1})	n
Volcanic sand	1.60	2.29	0.493	4.94
Fine sand	4.10	3.70	0.207	7.74
Touchet silt loam	7.50	1.82	0.101	4.00
Foxhill sandstone (1)	1.03	1.92	0.743	4.21

absolute value S of the slope of S_w with log h is given by

$$S = \left|\frac{dS_w}{d(\log h)}\right| = \frac{1}{(\theta_s - \theta_r)}\left|\frac{d\theta}{d(\log h)}\right| = \frac{1}{(\theta_s - \theta_r)}\ln(10)h\left|\frac{d\theta}{dh}\right| \tag{5}$$

Differentiating Eq. (4a) and recasting,

$$\left|\frac{d\theta}{dh}\right| = \frac{\tau}{h}(\theta_s - \theta_r)S_w \tag{6}$$

From Eqs. (5) and (6),

$$S = 2.303\,\tau\,S_w \tag{7}$$

According to Van Genuchten, the best location on the retention curve for evaluating the slope S is halfway between θ_s and θ_r. At this halfway point P,

$$\theta_p = (\theta_s + \theta_r)/2 \tag{8a}$$

$$h_p = h_e(0.5)^{-1/\tau} \tag{8b}$$

$$S_w = 0.5 \tag{8c}$$

The slope S_p of the BC retention curve at this point is, from Eq. (7),

$$S_p = 1.151\,\tau \tag{9}$$

Van Genuchten (1980) proposed the following equations for estimating his parameter m:

$$m = 1 - \exp(-0.8\ S_p), \qquad 0 < S_p \leqslant 1 \tag{10a}$$

$$m = 1 - \frac{0.5755}{S_p} + \frac{0.1}{S_p^2} + \frac{0.025}{S_p^3}, \qquad S_p > 1 \tag{10b}$$

Applying Eq. (2) at P,

$$\alpha = \frac{1}{h_p}(2^{1/m} - 1)^{1/n} \tag{11}$$

4.4. Application and results

As an example, BC model parameters were calculated for four soils of varying

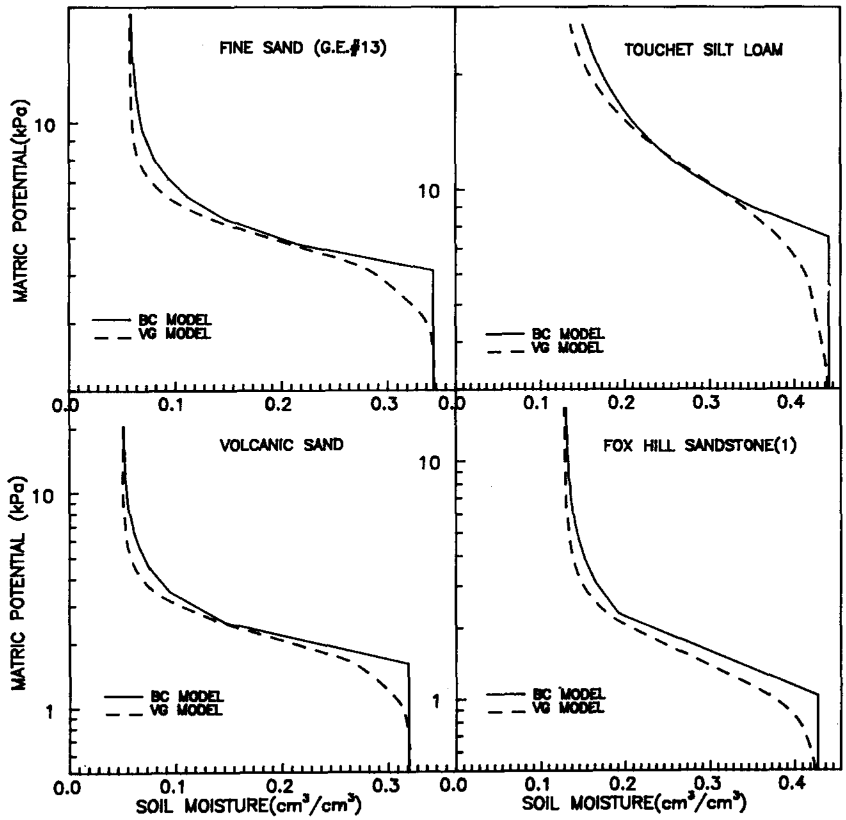

Fig. 4. Fig. 4. Comparison of given (Brooks and Corey, 1964) BC model and estimated equivalent VG model SMCs for four soils.

textures from the retention data given by Brooks and Corey (1964), and VG model parameters were derived therefrom using the above algorithm. Table 1 shows these parameters. The moisture characteristics of the soils, generated with the parameter sets given in Table 1, are graphically presented in Fig. 4.

For the sandy loam soil of the present study, the in situ SMC could be described by the BC model with the optimised parameters $h_e \doteq 2.066$ kPa and $\tau = 0.419$. The proposed conversion algorithm yielded equivalent VG model parameters of $\alpha = 0.37$ kPa^{-1} and $n = 1.47$. These values are close to the independent non-linear LS regression estimates of $\alpha = 0.4156$ kPa^{-1} and $n = 1.415$. Further, VG model estimates with only parameters α and n being modified to values obtained by the conversion procedure compare favourably with field retention data (Fig. 1) and field $K(\theta)$ data (Fig. 2).

5. Conclusions

For the sandy loam soil of the present study, the closed-form expression of

Van Genuchten (1980) provided estimates of unsaturated hydraulic conductivity (K) that are very close to field measurements obtained from an internal drainage test. The other commonly used closed-form model, that of Brooks and Corey (1964), overpredicted K by about one order of magnitude, in the wet range. However, differences between the two model predictions decrease considerably in the dry range. For the present soil, accurate prediction of K in the wet range from retention properties appears to depend on the choice of the closed-form model. This aspect assumes significance for the modelling of hydrologic processes of groundwater recharge and evaporation, the magnitudes of which are known to increase non-linearly with soil moisture content.

Given the fact that the Brooks–Corey model parameters can be more easily estimated from available texture-based regression models or regression analysis of retention data, it is suggested that equivalent Van Genuchten model parameters be estimated using the proposed parameter conversion algorithm.

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