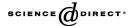


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Technical note

Hindcasting of storm waves using neural networks

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Abstract

Cyclone generated waves play a significant role in the design of coastal and offshore structures. Instead of conventional numerical models, neural network approach is used in the present study to estimate the wave parameters from cyclone generated wind fields. Eleven cyclones, which crossed the southern east coast of India between 1962 and 1979, are considered for analysis in this paper. The parametric hurricane wave prediction model by Young (1988) [Young, I.R., 1988. Parametric hurricane wave prediction model. Journal of Waterways Port Coastal and Ocean Engineering 114(5), 637–652] is used for hindcasting the wave heights and periods. Estimation of wave heights and periods is carried out using back propagation neural network with three updated algorithms, namely Rprop, Quickprop and superSAB. In neural network, the estimation is carried out using (i) difference between central and peripheral pressure, radius of maximum wind and speed of forward motion of cyclone as input nodes and the wave heights and periods as output nodes and (ii) wind speed and fetch as input nodes and wave heights and periods as output nodes. The estimated values using neural networks match well with those estimated using Young's model and a high correlation is obtained namely (0.99).

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Keywords: Hindcasting; Neural network; Storm waves; Back propagation; Wind speed; Fetch

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Nomenclature

ANN artificial neural network CC correlation co-efficient E global error function E_p error of pth pattern

 $I_2H_2O_2$ neural network structure with two input nodes (I_2), two hidden nodes (H_2)

and two output nodes (O_2)

 $I_3H_2O_2$ neural network structure with three input nodes (I_3), two hidden nodes (H_2)

and two output nodes (O_2)

IMD Indian Meteorological Department

NN neural network

 o_k weighted sum of the inputs of neuron i network output at kth output node P total number of training pattern

 s_i output of neuron i

 t_k target output at kth output node w_{ij} weight from neuron j to neuron i

YM Young's model

 η a factor

 Δ_{ij} update value of weight from neuron i to j Δw_{ij} weight increment from neuron i to j ε_{ij} sign-dependent learning-rate adaptation

 μ momentum factor

1. Introduction

Severe storms occur in Bay of Bengal and Arabian Sea generally during and after the monsoon. They give rise to very high seas, and during the summer monsoon seas are generally rough. The voluntary reporting ships do not note these high seas since ships usually avoid storms. Despite their obvious importance, the complex process active in their generation is only beginning to be understood.

Long-term wave data is necessary for computing extreme wave conditions or design wave statistics. As far as Indian seas are concerned recorded wave data is available for short periods for some places along the coasts. The spatial wave information of numerical wave forecasting schemes is useful and attractive in many applications, but it needs elaborate meteorological and oceanographic data and involves an enormous amount of computational effort. Hindcasting storm seas using past storm wind fields can overcome this deficiency.

Wave hindcasting is the prediction of waves based upon the past meteorological and oceanographic data. Wave hindcasting is extremely useful in the planning and maintenance of marine activities. Wave hindcasting is a non-real time application of numerical wave models in the broad field of climatology. Just as weather conditions,

the wave conditions will change from year to year, thus a proper statistical and climatological treatment requires several years of wave data and quite often from many locations simultaneously.

Wave hindcasting is useful for the design studies for harbours, coastal structures, offshore structures such as oil platforms, defense purposes, planning or operations, coastal erosion and sediment transport, environmental studies and wave energy estimation. Wave hindcasting calls for large amount of data and the computational effort involved is more. Also there is an element of uncertainty when wind fields are used for the hindcasting of waves.

Neural networks have the ability to recognize the hidden pattern in the data and accordingly estimate the values. Provision of model-free solutions, data error tolerance, built in dynamism and lack of any exogenous input requirement makes the network attractive. A neural network is an information processing system modeled on the structure of the human brain. Its merit is the ability to deal with fuzzy information whose interrelation is ambiguous or whose functional relation is not clear.

The use of neural network in the field of civil and ocean engineering is increasing. Some examples of the use of neural network in the civil engineering field are forecasting of rainfall (French et al., 1992), forecasting of runoff (Crespo and Mora, 1993), concrete strength (Kasperkiewicz et al., 1995). The uses of neural network in the coastal engineering are sea level prediction (Vaziri, 1997), estuarine instabilities (Grubert, 1995), stability analysis of rubble mound breakwaters (Mase et al., 1995), wave forecasting (Deo and Naidu, 1999; Rao et al., 2001) and tide prediction (Deo and Chaudhari, 1998; Mandal, 2001).

2. Neural network

The word Neural Network (NN) is used to normally describe the 'Artificial Neural Network' (ANN). Biological neural networks are much more complicated in their elementary structures than the mathematical models, which we use for ANN's. An ANN is a network of many very simple processors ('units'), each possibly having a (small amount of) local memory. The units are connected by unidirectional communication channels ('connections'), which carry numeric (as opposed to symbolic) data. The units operate only on their local data and on the inputs they receive through the connections. The easy adaptability to the solution is what distinguishes neural networks from other mathematical techniques. The feed forward neural networks will work as described below.

3. Feed forward neural networks

Normally in a feed forward network there will be three layers namely the input layer, the hidden layer and the output layer. The inputs to the network are given at the input layer. The number of input nodes would be equal to the number of input parameters. So the input nodes receive the data and pass them on to the hidden layer nodes. These nodes individually sum up the received values after multiplying each input value by a weight.

Then they attach a bias to this sum and pass on the result through non-linearity such as the sigmoid transfer function. This forms the input to the output layer that operates identically with the hidden layer nodes. Resulting transformed output from each output node forms the network output. In the present work the back propagation feed forward type network is used. The objective is to minimize the global error given as

$$E = \frac{1}{P} \sum E_p \tag{1}$$

and

$$E = 0.5 \sum (o_k - t_k^2) \tag{2}$$

where

P total number of training patterns;

 E_p error for pth pattern;

 o_k network output at kth output node;

 t_k target output at kth output node.

In the back propagation networks, the error between the target output and the network output is calculated and this will be back propagated using the steepest descent or gradient descent approach. The network weights and biases are adjusted by moving a small step in the direction of negative gradient of the error function during each iteration. The iterations are repeated until a specified convergence is reached. Due to the fixed step size it converges slowly and may exhibit oscillatory behavior and hence the back propagation networks with updated algorithms are used in the present paper. The algorithms used are Quickprop, Rprop and superSAB. A little description about the working of a back propagation neural network and algorithms is given below.

3.1. Back propagation learning

Back propagation is the most widely used algorithm for supervised learning with multilayer feedforward networks. The idea of the back propagation learning algorithm is the repeated application of the chain rule to compute the influence of each weight in the network with respect to an arbitrary error function E:

$$\frac{\partial E}{\partial w_{ii}} = \frac{\partial E}{\partial s_i} \frac{\partial s_i}{\partial \text{net}_i} \frac{\partial \text{net}_i}{\partial w_{ii}}$$
(3)

In this, w_{ij} is the weight from neuron i to neuron i, s_i is the output, and net i is the weighted sum of the inputs of neuron i. Once the partial derivative for each weight is known, the aim of minimizing the error function is achieved by performing a simple gradient descent:

$$w_{ij}(t+1) = w_{ij}(t) - \varepsilon \frac{\partial E}{\partial w_{ii}}(t)$$
(4)

3.2. Quickprop

The Quickprop algorithm is developed from the Newton's method, but it is more heuristic than formal. The method is similar to normal back propagation, but the variation is that for each weight this process keeps storing the error derivative computed during the previous training epoch, along with the difference between the current and previous values of this weight. The value for the current training epoch is also available at weight update time (Fahlman, 1988). The estimates of the position of the minimum for each weight are obtained by solving the equation given below

$$\Delta w_{ij}(t) = \frac{\frac{\partial E}{\partial w_{ij}}(t)}{\frac{\partial E}{\partial w_{ij}}(t-1) - \frac{\partial E}{\partial w_{ij}}(t)} \Delta w(t-1)$$
(5)

3.3. *Rprop*

Rprop stands for 'resilient propagation' and is an efficient new learning scheme that performs a direct adaptation of the weight step based on local gradient information. In crucial difference to previously developed adaptation techniques, the effort of adaptation is not blurred by gradient behavior whatsoever. To achieve this, they introduce for each weight its individual update value Δ_{ij} , which solely determines the size of the weight update. This adaptive update value evolves during the learning process based on its local sight on the error function E, according to the following learning rule: (Reidmiller and Braun, 1993).

$$\Delta_{ij}^{(t)} = \begin{cases}
\eta^{+} \Delta_{ij}^{(t-1)}, & \text{if } \frac{\partial E^{(t-1)}}{\partial w_{ij}} \frac{\partial E^{t-1}}{\partial w_{ij}} > 0 \\
\eta^{-} \Delta_{ij}^{(t-1)}, & \text{if } \frac{\partial E^{(t-1)}}{\partial w_{ij}} \frac{\partial E^{t-1}}{\partial w_{ij}} < 0 \\
\Delta_{ii}^{(t-1)}, & \text{else}
\end{cases}$$
(6)

where $0 < \eta^- < 1 < \eta^+$.

The adaptation rule works as follows. Every time the partial derivative of the corresponding weight w_{ij} changes its sign, which indicates that the last update was too big and the algorithm has jumped over a local minimum, the update value Δ_{ij} is decreased by the factor η . If the derivative is positive (increasing error), the weight is decreased by its update value, if the derivative is negative, the update value is added.

$$\Delta w_{ij}^{(t)} = \begin{cases} -\Delta_{ij}^{(t)}, & \text{if } \frac{\partial E^{(t)}}{\partial w_{ij}} > 0 \\ +\Delta_{ij}^{(t)}, & \text{if } \frac{\partial E^{(t)}}{\partial w_{ij}} < 0 \\ 0, & \text{else} \end{cases}$$

$$(7)$$

$$w_{ij}^{(t+1)} = w_{ij}^{(t)} + \Delta w_{ij}^{(t)} \tag{8}$$

3.4. SuperSAB

SuperSAB is based on the idea of sign-dependent learning-rate adaptation. The basic of the function is to change its learning-rate exponentially instead of linearly. This is done to take the wide range of temporarily suited learning-rates into account (Riedmiller, 1994).

In case of a change in sign of two successive derivatives, the previous weight is reversed. SuperSAB is considered to be fast converging algorithm. One of the problem with SuperSAB is the large number of parameters that need to be determined in order to achieve good convergence times, i.e. the initial learning-rate, the momentum factor, and the increase (or decrease) factors. Also one more drawback is inherent to all learning-rate adaptation algorithms, is the remaining influence of the size of the partial derivative on the weight step.

$$\Delta w_{ij}(t) = -\varepsilon_{ij}(t) \frac{\partial E}{\partial w_{ij}}(t) + \mu \Delta w_{ij}(t-1)$$
(9)

4. Young's model

The storm field variables namely central pressure, radius of maximum wind, storm speed, direction of movement of storm, latitude and longitude are extracted for each cyclone from the isobaric charts available with the archives of the Indian Meteorological Department (IMD) at 3 or 6 h time intervals. The method based on standard hydrometer pressure profile, presented by Varkey et al. (1996), is used for the hindcast of storm wind fields.

Young's model (YM) is used in the estimation of wave characteristics for the cyclones considered (Young, 1988). The input parameters to the model are radius of maximum wind, the maximum wind speed and the speed of forward motion. Using the JONSWAP fetch-limited growth relationship, the model estimates the maximum significant wave height and the spectral peak period of the maximum waves in the storm.

5. Results and discussions

The 11 dominant tropical cyclones crossed East Coast of India from 1962 to 1979 are considered in the present study. The cyclones considered are:

- C1 27–29 November 1962
- C2 20-23 December 1964
- C3 31 December 1965–3 January 1966
- C4 29 April-1 May 1966
- C5 06-07 December 1967
- C6 17–22 November 1972
- C7 28-31 October 1977
- C8 08–12 November 1977

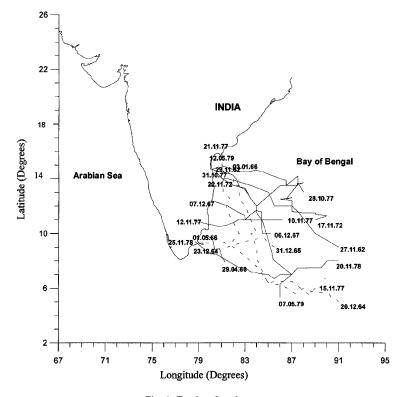


Fig. 1. Tracks of cyclones.

C9 15–21 November 1977 C10 20–27 November 1978 C11 07–12 May 1979

The tracks of the cyclones are presented in Fig. 1. Out of the above 11 cyclones C2, C3 and C9 cyclone data are considered to train the neural network. The remaining eight cyclones data is used for testing the neural network.

The professional version of the backprop NN software developed by Tveter (2000) is used in the present study. This software supports various update algorithms namely Rprop, Quickprop, superSAB which are used for this work.

For all the cyclones considered in this study, various parameters of winds and waves are available in the technical report (Sanil Kumar et al., 2001). First the estimation of wave parameters is carried using the radius of maximum wind, speed of forward motion of cyclone and central pressure as inputs to the neural networks. The outputs are the significant wave height and spectral peak period. Next the input parameters are changed to wind speed and fetch and again the estimation is carried out to obtain the wave heights and periods. So, for the first estimation the NN structure used is $I_3H_2O_2$ where I_3 , three input neurons (difference between central and peripheral pressure, radius of maximum wind

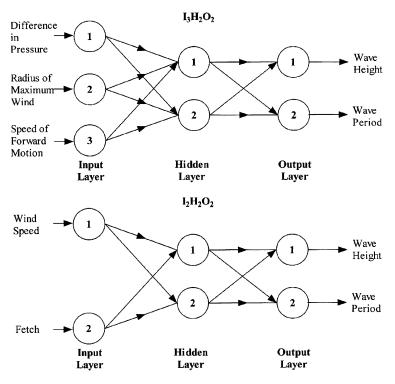


Fig. 2. Neural network structures.

and speed of forward motion), H_2 , two hidden neurons and O_2 , two output neurons (wave height and wave period). For the second one the NN structure used is $I_2H_2O_2$, where I_2 , two input neurons (wind speed and fetch), H_2 , two hidden neurons and O_2 , two output neurons (wave height and wave period). The network structures used in the present study are shown in Fig. 2.

5.1. Comparison of wave heights and periods estimated by NN with YM for the cyclone C1 (27–29 November 1962)

The storm variables such as central pressure, radius of maximum wind and storm speed are estimated from isobaric charts available for 3 h interval. The input parameters considered are the difference between central and peripheral pressure, radius of maximum wind and speed of forward motion $(I_3H_2O_2)$. The wave heights and periods estimated using the neural networks $(I_3H_2O_2)$ are presented in Fig. 3 together with those estimated using Young's model as a function of time. The correlation coefficients for wave height and wave period are also presented in the figure. The results obtained using the three update (Quickprop, Rprop, SuperSAB) algorithms are presented in the same figure. The NN estimated values match almost exactly with those estimated using the Young's model. A very high correlation coefficient of above 0.99 is obtained with Rprop and SuperSAB

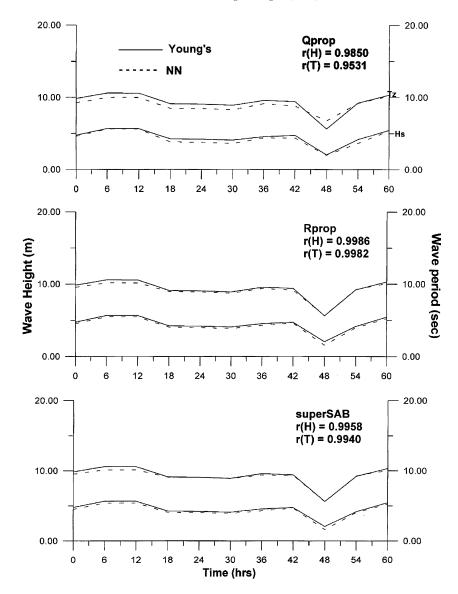


Fig. 3. Estimation of wave heights and periods of November 1962 cyclone (I₃H₂O₂).

and with Quickprop it is slightly less. The maximum wave height and period obtained from Rprop (5.48 m, 10.162 s) matches reasonably well with those estimated from Young's model (5.68 m, 10.59 s). The percentage deviation from Young's model estimated data is 3% for wave height and 4% for wave period.

The wave heights and periods are also estimated using the maximum wind speed and fetch as inputs $(I_2H_2O_2)$. The results obtained using the three update algorithms are shown in Fig. 4. The rise and fall tendencies are promptly picked up by NN. The values are almost

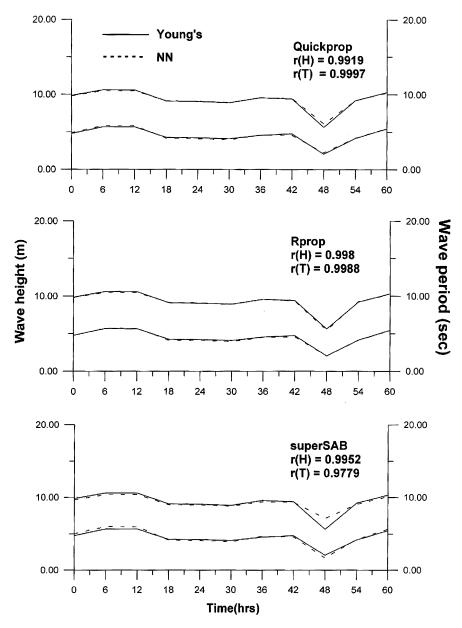


Fig. 4. Estimation of wave heights and periods of November 1962 cyclone (I₂H₂O₂).

matching when Quickprop and Rprop are used but there is a slight deviation from the Young's model estimated values when SuperSAB is used which can be seen from the figure. Estimation using this set of input parameters $(I_2H_2O_2)$ gave a better correlation than the previous one $(I_3H_2O_2)$. This may be due to the more random nature of waves and

the complex relationship between the input and output parameters. The maximum wave height and its period estimated using Rprop (5.72 m, 10.46 s) closely match with those estimated using Young's model (5.68 m, 10.59 s). The percentage deviation from Young's model estimated data is 0.6% for wave height and 1.22% for wave period.

Similar comparisons are made for other storms also. It is found that the significant wave heights and spectral peak periods estimated using neural networks closely match with

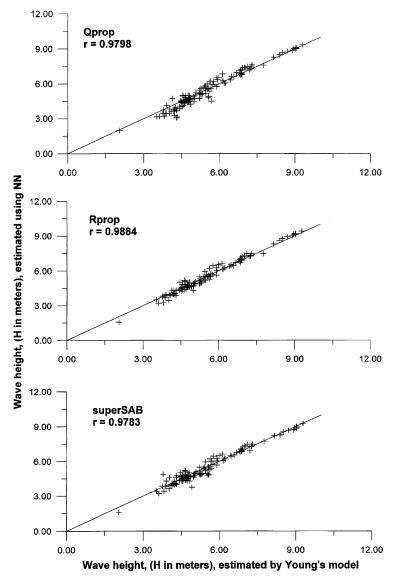


Fig. 5. Correlation between wave heights estimated using NN and Young's Model (I₃H₂O₂).

those estimated using Young's model. The percentage variation of wave heights between Young's model and NN data is observed to be between 0.3 and 3% and for wave period is 0.2 and 4%.

5.2. Correlation between wave heights estimated using YM and NN

The correlation between the estimated wave heights by NN and by Young's model for network structure $I_3H_2O_2$ is shown in Fig. 5. A little deviation from the best-fit line is observed when Quickprop algorithm is used and with Rprop and SuperSAB the scatter is very less. The high correlation coefficient obtained confirms this. Table 1 gives the comparison between the correlation coefficients obtained for wave heights using the three algorithms for both the cases considered for $I_3H_2O_2$. From the table it can be seen that the correlations obtained when Rprop is used are higher than the correlations obtained with the other two algorithms. Similarly the correlation between NN estimated wave heights and Young's model estimated wave heights are presented in Fig. 6 for the $I_2H_2O_2$ network structure. The scatter observed is somewhat more when SuperSAB is used but with the other two algorithms the values are almost on the best-fit line. Table 1 also gives the comparison between correlation coefficients obtained for various cyclones with the three algorithms for $I_2H_2O_2$.

5.3. Correlation between wave periods estimated using YM and NN

The correlation between the wave periods estimated using NN and Young's model is shown in Figs. 7 and 8. From the figures it can be seen that there is some scattering when the Quickprop algorithm is used. The scatter is not much for the best fit lines when the other two algorithms (Rprop, superSAB) are used. The correlation coefficient obtained also confirms this. Table 2 gives the comparison between the correlation coefficients obtained for wave periods using the three algorithms for both the cases considered. From the table it can be seen that the correlations obtained when superSAB algorithm is used, are higher than the correlation obtained with other two algorithms.

Table 1				
Comparison of correlation	coefficients	obtained	for wave	heights

Cyclone	$I_3H_2O_2$			$I_2H_2O_2$		
	Quickprop	Rprop	SuperSAB	Quickprop	Rprop	SuperSAB
C1	0.9850	0.9986	0.9958	0.9919	0.9980	0.9952
C4	0.9950	0.9521	0.9464	0.9952	0.9948	0.9949
C5	0.9161	0.9947	0.9947	0.9534	0.9865	0.9422
C6	0.9570	0.9759	0.9751	0.989	0.9964	0.9862
C7	0.9752	0.9400	0.9474	0.9888	0.9855	0.9901
C8	0.9847	0.9925	0.9932	0.9966	0.9928	0.9956
C10	0.9651	0.9835	0.9857	0.9914	0.9976	0.9823
C11	0.9959	0.9947	0.9946	0.9961	0.9995	0.9957

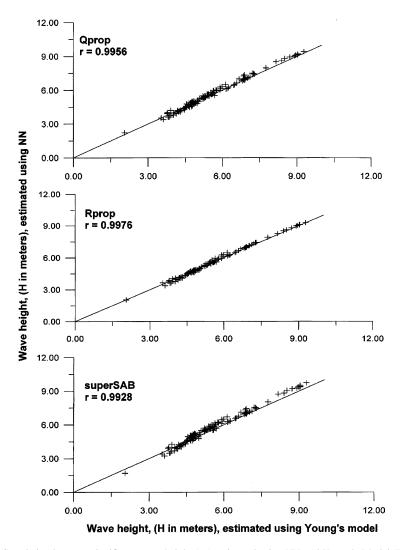


Fig. 6. Correlation between significant wave height (H_s) estimated using NN and Young's Model (I₂H₂O₂).

5.4. Joint distribution for YM and NN estimated data

The correlation between wave heights and periods estimated using NN and YM are presented in Fig. 9 for $I_3H_2O_2$ and Fig. 10 for $I_2H_2O_2$. From Fig. 9, it can be observed that for wave heights greater than 4.5 m the NN estimated periods are on the lower side of YM estimated periods whereas for wave heights less than 4.5 m the NN estimated periods are on the higher side of YM estimated periods. However, the estimated NN values are almost on the best-fit line but a bit of scatter is noticed when Quickprop is used. From Fig. 10, it can be clearly seen that the best fit lines for both NN estimated values and Young's model estimated values are very close and the data points are matching well.

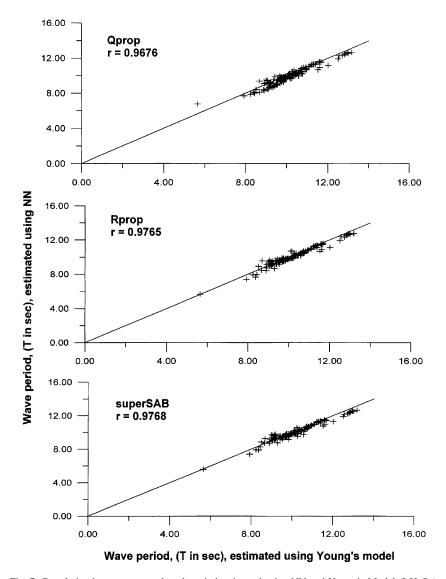


Fig. 7. Correlation between spectral peak period estimated using NN and Young's Model (I₃H₂O₂).

6. Conclusions

Based on the present investigation the following conclusions are drawn

- The significant wave heights and spectral peak periods estimated using neural networks closely match with those estimated using Young's model.
- The very high correlation coefficient of about 0.99 obtained in all cases confirms that neural networks can be effectively used for the cyclonic wind waves estimation.

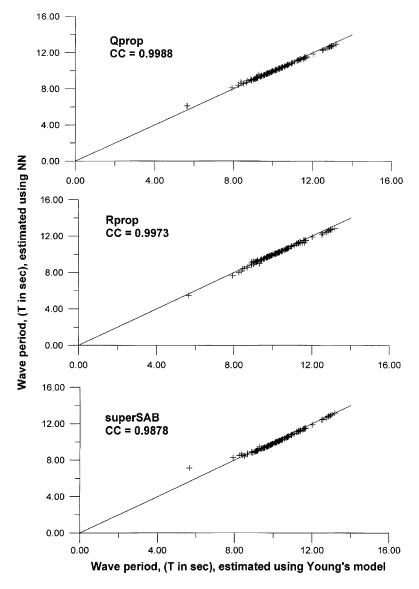


Fig. 8. Correlation between spectral peak period estimated using NN and Young's Model (I₂H₂O₂).

- The percentage variation of wave heights between Young's model and NN data is observed to be between 0.3 and 3% and for the wave periods is 0.2 and 4%.
- Joint distribution obtained with NN is having the same trend with that of Young's model data.
- Wave heights and periods estimated using wind speed and fetch as input parameters match more closely with Young's model values than with those estimated using

Table 2 Comparison of correlation coefficients of wave periods for $I_3H_2O_2$

Cyclone	$I_3H_2O_2$			$I_2H_2O_2$		
	Quickprop	Rprop	SuperSAB	Quickprop	Rprop	SuperSAB
C1	0.9531	0.9982	0.9940	0.9997	0.9988	0.9779
C4	0.8836	0.9665	0.9714	0.9999	0.9990	0.9995
C5	0.9628	0.9516	0.9520	0.9972	0.9850	0.9998
C6	0.9416	0.9441	0.9459	0.9994	0.9949	0.9904
C7	0.9517	0.9077	0.9467	0.9998	0.9997	0.9994
C8	0.9959	0.9911	0.9917	0.9996	0.9990	0.9995
C10	0.9478	0.9399	0.9822	0.9962	0.9982	0.9935
C11	0.9738	0.9388	0.9808	0.9979	0.9973	0.9978

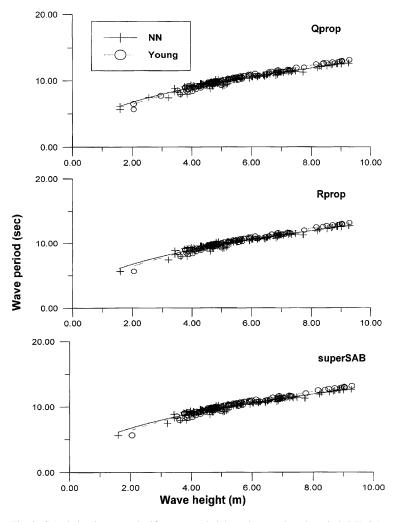


Fig. 9. Correlation between significant wave height and spectral peak period (I₃H₂O₂).

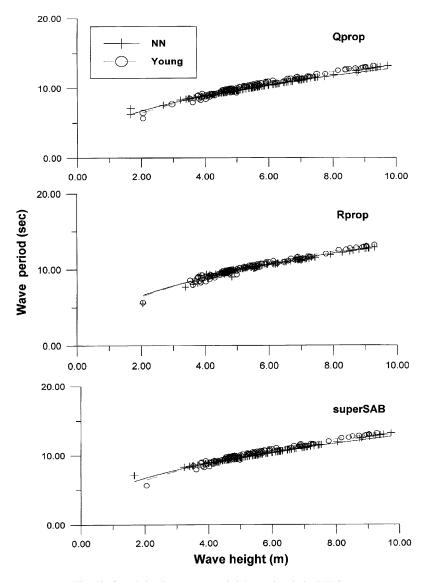


Fig. 10. Correlation between wave heights and periods (I₂H₂O₂).

the difference in pressure, speed of forward motion of cyclone and radius of maximum wind as inputs.

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