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Non-orthogonal space–frequency block codes from cyclic codes for wireless systems employing MIMO-OFDM with index modulation

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ABSTRACT

Space–frequency codes (SFC) having error correcting structure can be used to enhance the bit error rate (BER) performance of modern wireless systems (5G and beyond) employing multiple-input multiple-output (MIMO) and multi-carrier communication. In this work, the construction of non-orthogonal space–frequency block codes (NSFBC) from (n, k) cyclic codes has been proposed. In which, n represents the number of symbols in the codeword and k represents the number of symbols in the information sequence. The performance of proposed codes has been evaluated in MIMO systems employing orthogonal frequency division multiplexing and index modulation (MIMO-OFDM-IM). We initially obtained (n, k) full rank cyclic codes for any $1 < k < \lfloor \frac{n}{m} \rfloor$ using Galois field Fourier transform (GFFT) description of (n, k) cyclic codes over $\mathbb{F}_{q^m}$. Further, NSFBCs are obtained from full rank codes using Rank preserving maps. In a 2×2 system and a 10-path MIMO channel, the proposed full rank NSFBC with rank-preserving IM mapping (FR-NSFBC-IM), over $\mathbb{F}_{5^2}$, provides he similar BER performance when compared to MIMO-OFDM-IM system with Rate-1 Alamouti code and QPSK. Moreover, it provides an improvement in spectral efficiency of about 0.9 b/s/Hz. When compared to the MIMO-OFDM-IM with BPSK, FR-NSFBC-IM codes over $\mathbb{F}_{5^2}$ provide an asymptotic SNR gain of about 1 dB and also the spectral efficiency has been improved by about 0.6 b/s/Hz. In the 4×4 scenario, full rank NSFBCs over $\mathbb{F}_{5^4}$ with rank deficient IM mapping (RD-NSFBC-IM) provide an improvement in spectral efficiency of about 1.3 b/s/Hz. However, BER performance is similar to that of MIMO-OFDM-IM with BPSK.

1. Introduction

Attainment of high data rates with high levels of information integrity is the most significant challenge being addressed by modern multi-carrier wireless communication technologies such as IEEE 802.11n, IEEE 802.16e, Long Term Evolution (LTE 4G), and 5G and beyond [1]. Though OFDM could not give best solution while producing the required data rates and information integrity levels, due to its requirement of higher degree levels for synchronization, it is still in contention as one of the waveform alternatives [2]. Hence, multiple-input multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) based communication has been greatly addressed in modern wireless communications, to attain the high data rates along with high levels of integrity. Recently, with the intervention of index modulation in MIMO-OFDM (MIMO-OFDM-IM), similar data rates as that of MIMO-OFDM have been obtained. Moreover, less number of carriers are sufficient in the case of MIMO-OFDM-IM when compared to traditional MIMO-OFDM at any instance of time [1]. Hence, MIMO-OFDM-IM communication has been considered as one of the other alternatives to MIMO-OFDM communication scheme. In MIMO-OFDM-IM, additional degree of freedom offered by the choice of subcarriers for modulation (active subcarriers) is exploited for information transmission [3,4]. The information is directly modulated onto the chosen subcarriers once the selection of active subcarriers is done. From the theory of MIMO communications and MIMO-OFDM communications, one can understand that the performance (in terms of reliability) of MIMO-OFDM-IM can be enhanced using space–time block codes—STBCs (in case of SM-MIMO-OFDM systems) or space–frequency block codes—SFBCs (in case of MIMO-OFDM-IM system) [5]. STBCs and SFBCs based on Rank-1 Alamouti code with different capabilities have been designed and evaluated in previous works of MIMO-OFDM [6–9] and Spatial modulation based MIMO systems [10].

The performance of MIMO based communication system is further improved by using an (n, k) error control code in concatenation with the STBC [11]. Where, n is the number of output symbols, and k is the number of input information symbols of an encoder. However, it needs an additional error control encoder and decoder which further increases computational complexity. Hence, there is a need in achieving better performance in STBC

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or SFBC based MIMO communications without using an external error control code. This can be solved by introducing STBC or SFBC based MIMO communications with internal error correcting structure. A few works have already been proposed [12–15] in this aspect and the details of the existing approaches are as given below:

1. STBCs with rank $r < N_T$, obtained from Bose–Chaudhuri–Hocquenghem (BCH) and Reed–Solomon (RS) codes, were used in [12] for $N_T \times 4$ MIMO communications (where, N_T represents the number of transmit antennas). Here, performance improvement is obtained by maximizing the minimum squared Euclidean distance between any two codewords. Hamming metric (not a rank metric) has been considered to design STBCs resulting in STBCs with rank $r < N_T$. Due to this, at least four receive antennas are used to satisfy the condition $rN_R \geq 4$. This limits the usage of codes to MIMO systems employing $N_R \geq 4$ receive antennas. Hence, there is a need for full rank STBCs for MIMO systems with $N_R < 4$ antennas. The problem of synthesizing full rank STBCs ($rank = N_T$) with inherent error correcting structure has been partially addressed in [13–15].
2. Gabidulin codes over $\mathbb{F}_{q^m}$ (along with rank preserving Gaussian integer map) have been used by [13] as STBCs for MIMO systems. The codewords of a Gabidulin code are viewed as $m \times n$ matrices over the base field $\mathbb{F}_q$. These constructions ensure that $n \leq m$. Further, they are extended to a class of cyclic codes, also called $q-$cyclic codes. Because of the maximum rank distance property and the code structure which ensures that $n \leq m$, Gabidulin codes can be used as STBCs (full rank) if $n = m$ and $d = m$, implying $k = 1$. Thus, $(n, 1)$ Gabidulin codes alone with $n = m$ are used as STBCs, resulting in obtaining q^m possible STBC codewords.
3. In [14], $(n, 1)$ full rank codes obtained using discrete Fourier transform (DFT) description were used as STBCs in MIMO systems. It has been shown that the proposed STBCs outperform their orthogonal counterpart in the case of quasi-static Rayleigh fading environment. The construction involves DFT description of cyclic codes over $\mathbb{F}_{q^m}$ [15]. Unlike Gabidulin codes, the length n of the code is chosen to be a divisor of $q^m - 1$, hence codewords of length $n \geq m$ are possible. Similar to Gabidulin codes, the codewords of these codes can be viewed as $m \times n$ matrices over the base field $\mathbb{F}_q$, with $n \geq m$. The separation (in terms of rank distance) between any two codewords is found to be at least d, with $d \leq m$; and when $d = m$ full rank codes are obtained. However, as per the analysis given in [15] rank of the cyclic code remains full (rank $= m$), if the code is constructed using only one free transform domain component (say A_j), where the index j is coming from $q-$cyclotomic coset of size $e_j = m$, resulting in $(n, 1)$ codes.

The codes presented in [12–15] are $(n, 1)$ codes. However, from coding theory, we know that the error performance of a code can be improved with the increase in d ($d \leq n - k + 1$) by maintaining same code rate k/n. Hence, there is a need to synthesize full rank codes with higher k and maximum Hamming distance d. The problem of synthesizing full rank codes for $k \geq 2$ has not been addressed in [12–15] and also in earlier literature. Motivated by the requirement of (n, k) full rank codes with $k \geq 2$ for MIMO communications, in this paper, we have explored the possibility of designing SFBCs from $(n = km, k)$ full rank codes over $\mathbb{F}_{q^m}$, with $k \geq 2$. The concept of Galois field Fourier transform (GFFT) approach has been used for the following reasons.

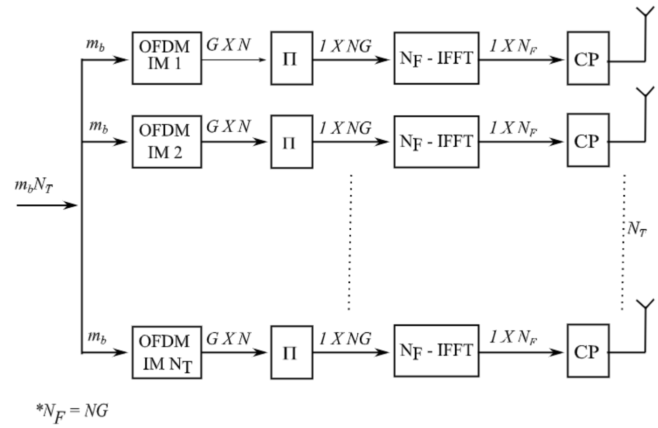

Fig. 1. Block-Diagram of conventional $N_T \times N_R$ MIMO-OFDM-IM Transmitter [4].

- It allows us to draw direct relationships between the size of a $q-$cyclotomic coset and the rank of a cyclic code.
- The choice of free transform components give an additional degree of freedom, that helps in grouping few full rank codes into one NSFBC (as shown in Section 3.2.3).

The novelty and major contributions of this paper are as follows:

1. Analysis of the rank-distance properties of (n, k) cyclic codes over $\mathbb{F}_{q^m}$ and construction of (n, k) full rank codes with $k \geq 2$.
2. The design of non-orthogonal space–frequency block codes (NSFBC) from full rank codes and their performance evaluation in MIMO-OFDM-IM systems.
3. Derivation of a theoretical upper bound on the BER performance of the proposed NSFBCs over frequency selective channels.

The rest of the paper is organized as follows: In Section 2, we have briefly described the MIMO-OFDM-IM system given by Basar [4]. The proposed NSFBC based MIMO-OFDM-IM system is discussed in Section 3. In this Section, we first explain the design of full rank codes from (n, k) cyclic codes, with all the preliminaries required (Section 3.2). We have stated theorems that describe the rank distance properties of cyclic codes for $k \geq 2$. Following this, we discuss the rank distance properties of (n, k) cyclic codes. This is followed by the explanation on the use of full rank codes as NSFBCs by utilizing rank preserving maps. Additionally, we have also described the formation of NSFBC codebook (Section 3.2.3). Section 3.3 discusses the application of the proposed NSFBCs in MIMO-OFDM-IM systems. Furthermore, we have provided details of the achievable spectral efficiency and computational complexity of the receiver. Section 4 provides the theoretical upper bound on the BER performance of the NSFBC-MIMO-OFDM-IM system. In Section 5, we provide the simulation results over time-flat frequency selective channels. The conclusions made out of this work are given in Section 6, with the summary of the various contributions made. Finally, the proofs of the proposed constructions are provided in Appendix

Throughout this paper, *bold* and *small* letters represent row vectors; *bold* and *capital* letters in italic represent column vectors; *bold* and *capital* letters represent matrices. The list of various symbols employed in this paper is provided in Table 1.

2. MIMO-OFDM-IM revisited

Index modulation based MIMO-OFDM communication system (MIMO-OFDM-IM) is proposed by Basar in [4]. The block diagram of MIMO-OFDM-IM transmitter is depicted in Fig. 1 [4].

Table 1
The symbols used in this article along with proper description.

$\mathbb{F}_q$—finite field of characteristic q.	N_T—Number of transmit antennas.
n—Length of the code (codewords)	N_R—Number of receive antennas.
m—Smallest integer such that $n \| q^m - 1$	N—Number of orthogonal subcarriers per $\mathbf{X}_\rho$.
$\mathbb{F}_{q^m}$—mth extension field of $\mathbb{F}_q$	N_F—Total number OFDM carriers
α—Primitive $(q^m-1)^{\text{th}}$ root of unity in $\mathbb{F}_{q^m}$.	$\boldsymbol{Y}$—Received column vector corresponding to. NSFBC-IM codeword
$\mathscr{C}$—Cyclic code of length n.	$\mathbf{Y}$—Received Matrix.
$\mathbf{c}$—n-length codeword vector $\in \mathscr{C}$	$\mathbf{H}$—Channel matrix.
$\mathbf{C}$—$m \times n$ matrix corresponding to $\mathbf{c}$	η—Spectral efficiency
$\mathbf{X}_\rho$—NSFBC codeword obtained from $\mathbf{C}$.	$\|\|[\cdot]\|\|_F^2$—Frobenius norm or L_2-norm.
$\mathbf{X}_{\rho,IM}$—NSFBC-IM codeword.	$[]_n$—q-cyclotomic coset mod n.
$\mathcal{X}$—NSFBC obtained from $\mathscr{C}$.	$\mathcal{L}$—Number of q-cyclotomic cosets of size m
$\mathbf{S}$—NSFBC-MIMO-OFDM-IM block .	L—Length of the channel impulse response.
$R_q(\cdot)$—Rank of $[\cdot]$.	

Considering that the system has N_T transmit antennas, a total of m_bN_T bits is taken as input for transmission. Corresponding to N_T transmitter branches, the input m_bN_T bits are further split into N_T groups of m_b bits each. At each OFDM-IM block, each group of m_b bits are further split into G subgroups. Each subgroup contains p bits that are further divided into p_1 data bits and p_2 selection bits. The p_1 data bits are mapped onto complex symbols of $M-ary$ constellation under consideration, while p_2 selection bits are used for index modulation. Thus, the output of each subgroup is an $N \times 1$ index modulated subblock. Such G subblocks (corresponding to G subgroups) are stacked and interleaved to form OFDM-IM frame of size $1 \times N_F$. The inverse fast Fourier transform (IFFT) block at the transmitter branch then processes this $1 \times N_F$ OFDM-IM frame. At each branch of the transmitter, one OFDM-IM frame is generated and transmitted using corresponding transmit antenna. As there are N_T transmit antennas, N_T OFDM-IM frames are transmitted through the channel simultaneously.

In the above system, the information (i.e. data bits) is directly modulated onto the orthogonal subcarriers. However, from the theory of MIMO communication, it has been shown that the performance can be further enhanced using block codes like STBC or SFBC [5]. In this work, we propose the use of NSFBC which is derived from (n, k) cyclic codes. The construction process and usage of NSFBCs with MIMO-OFDM-IM system are detailed in the subsequent section.

3. NSFBC based MIMO-OFDM-IM system

3.1. General case

The general block diagram of the proposed NSFBC-MIMO-OFDM-IM system is shown in Fig. 2. Initially, the incoming information is divided into data bits (Db) and carrier selection bits (Csb). Unlike conventional MIMO-OFDM-IM, in the proposed scheme, the data bits are mapped onto a $N_T \times e_j$ block code. The carrier selection bits are then used by index modulator to assign e_j subcarriers (out of available N OFDM carriers per codeword) to each row of NSFBC codeword. It produces $N_T \times N$ index modulated NSFBC (NSFBC-IM) codeword. The $N_T \times N$ NSFBC-IM codeword is then passed through OFDM modulator consisting of N_T $N-$ point IFFT blocks. Each IFFT block outputs $N-$ point IFFT vector of the corresponding row of NSFBC-IM codeword. N_T IFFT vectors at the output modulator are then transmitted using N_T antennas. The construction of NSFBCs that are obtained from cyclic codes are explained in next section.

3.2. Design of non-orthogonal SFBCs

This section discusses the rank distance properties of (n, k) cyclic codes over $\mathbb{F}_{q^m}$ along with the usage of full rank cyclic codes as NSFBCs. Here, we used rank preserving maps for obtaining NSFBCs. We show that one or more full rank codes can be grouped together to obtain a composite code containing large number of codewords. Each codeword is of full rank with respect to each other. This facilitates the formation of NSFBC codebook. The details of the MIMO-OFDM-IM system employing the proposed composite NSFBCs are given in Section 3.3.

3.2.1. Preliminaries of full rank codes

- **Cyclotomic Cosets**
For any positive integer $j \in [0, n-1]$, the $q-$ cyclotomic coset of j mod n is defined as
$$[j]_n = \left\{j, jq, \ldots, jq^l, \ldots, jq^s, \ldots, jq^{l+s}, \ldots, jq^{r_j-1}\right\}$$
the cardinality of $[j]_n$ is denoted as r_j.
- **Separation of q− Cyclotomic Coset Coefficients**
Let $[j]_n = \left\{j, jq, \ldots, jq^l, \ldots, jq^s, \ldots, jq^{l+s}, \ldots, jq^{r_j-1}\right\}$ be the $q-$ cyclotomic coset of integer j. We define the Separation between two elements jq^l and jq^{l+s} as the difference in powers of q associated with the elements i.e. Separation between jq^l and jq^{l+s}, is $l+s-l=s$.
- **Reciprocal Polynomial**
If $f(x)$ is a minimal polynomial (irreducible) given by $f(x) = a_tx^t + a_{t-1}x^{t-1} + \cdots + a_1x + a_0$ then the polynomial
$$f(x)^* = x^t * f(x^{-1}) = a_0x^t + a_1x^{t-1} + a_2x^{t-2} + \cdots + a_{t-1}x^1 + a_t;$$
is called the reciprocal polynomial of $f(x)$ Note that the degree of $f(x)^*$ is same as that of $f(x)$ with coefficients positioned in reverse order. Also, if $f(x)$ is irreducible then $f(x)^*$ is also irreducible [16].
- **Galois field Fourier transform (GFFT)**
In coding theory, Galois field Fourier transform (GFFT) is defined as DFT over Galois fields, with a primitive element α [17]. Following [17], for the case $n|q^m-1$ the GFFT of a vector $\mathbf{u} = \{u_i,\ 0 \le i \le n-1\}$, $u_i \in \mathbb{F}_{q^m}$ is defined as [17].
$$U_j = \sum_{i=0}^{n-1} u_i\beta^{-ij}\ ; \qquad 0 \le j \le n-1 \tag{1}$$
Here, $\beta = \alpha^{\frac{q^m-1}{n}}$ is the nth root of unity in F_{q^m}. The inverse GFFT (IGFFT) of $\mathbf{U} = \left\{U_j,\ 0 \le j \le n-1\right\}$ is given by,
$$u_i = (n \bmod q)^{-1} \sum_{j=0}^{n-1} U_j\beta^{-ij}\ ; \qquad 0 \le i \le n-1 \tag{2}$$
Following usual terminology, $\mathbf{u} = (u_0, u_1, \ldots, u_{n-1})$ is called the time domain vector and $\mathcal{U} = (U_0, U_1, \ldots, U_{n-1})$ is called the transform domain vector of $\mathbf{u}$.
In this paper, we use (2) to construct the full rank codes.
- **Rank**
Let $\mathbf{u} = (u_0, u_1, u_2, \ldots, u_{n-1})$ with $u_i \in \mathbb{F}_{q^m}$, $0 \le i \le n-1$. Since each u_i can be expressed as $m-$ tuple over $\mathbb{F}_q$, the vector $\mathbf{u}$ can be written as $m \times n$ matrix obtained by expanding each element of $\mathbf{u}$ as an $m-$tuple along a basis of $\mathbb{F}_{q^m}$ over $\mathbb{F}_q$. Such a matrix is shown in (3).

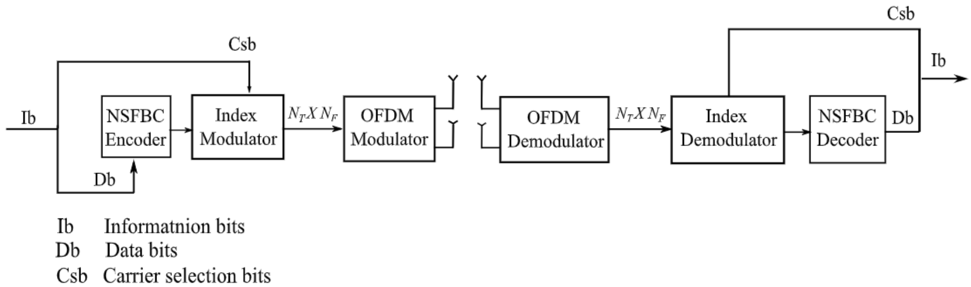

Fig. 2. Block-Diagram of NSFBC based $N_T \times N_R$ MIMO-OFDM-IM system.

$$\begin{bmatrix} u_{0,0} & u_{0,1} & u_{0,2} & \cdots & u_{0,n-1} \\ u_{1,0} & u_{1,1} & u_{1,2} & \cdots & u_{1,n-1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ u_{m-1,0} & u_{m-1,1} & u_{m-1,2} & \cdots & u_{m-1,n-1} \end{bmatrix} \quad (3)$$

Rank of $\mathbf{u}$ ($R_q(\mathbf{u})$) is now defined as the rank of the above matrix over $\mathbb{F}_q$.

If $R_q(\mathbf{u}) = m$ (number of rows), then we call the above matrix as full rank.

3.2.2. Rank distance properties of (n, k) cyclic codes

In coding theory, GFFT description can be used to design cyclic codes. Based on reference to (2), each value of U_j at particular j results in a distinct time domain vector $\mathbf{u}$. Since $U_j \in \mathbb{F}_{q^m}$ $(0 \leq j \leq n-1)$, there are q^{mn} time domain vectors corresponding to q^{mn} possible transform domain vectors.

If $\mathscr{C}$ is any linear code with codeword vectors $\mathbf{c} = \mathbf{u}$ (codeword matrices $\mathbf{C} = \mathbf{U}$) having elements in $\mathbb{F}_{q^m}$, then rank of $\mathscr{C}$ is defined as the minimum rank of the codeword vectors (codeword matrices over the base field $\mathbb{F}_q$) of $\mathscr{C}$. If $R_q(\mathscr{C}) = m$, then $\mathscr{C}$ is said to be full rank code.

In [15], it has been shown that the rank distance properties of cyclic code can be analyzed by choosing k free transform components with indices j, in a particular manner and confining the other $n-k$ transform components to zero. It is also mentioned that the $(n, 1)$ codes, over $\mathbb{F}_{q^m}$, having rank m can be constructed. Further, it is stated that $R_q(C)$ drops to less than m, if more than one value of j is chosen from the same $q-$ cyclotomic coset $[j]_n$. In specific, the exact rank of $(n, 2)$ codes with the two free transform component indices that are chosen from the same $q-$ cyclotomic coset has been determined. However, the approach given in [15] did not provide a closed form expression for the exact rank of (n, k) codes that are designed with $k \geq 3$. Also, the rank distance of $\mathscr{C}$ obtained by choosing k free transform components, with the index of each component selected from different $q-$ cyclotomic coset, is not focused. In this paper, we show that the full rank codes can be obtained for $k \geq 2$ and discuss the rank distance properties of $(n, k \geq 3)$ codes.

Now onwards, the notation $\mathbf{c}$ will be used to represent codeword vectors of $\mathscr{C}$.

Lemma 1. *If α_1 is root of a minimal polynomial y_1 of degree r_1 and α_2 is a root of minimal polynomial y_2 of degree r_2, then the degree e_j of the minimal polynomial for which both α_1 and α_2 are roots is*

- $e_j = r_1$; if $y_1 = y_2$
- $e_j = r_1 + r_2$; if $y_1 \neq y_2$

Proposition 1. *Let $\mathscr{C}$ be a cyclic code which is obtained using free transform components $\{U_{j_1}, U_{j_2}, \ldots, U_{j_k}\}$. Let $\beta^{[j_1]}, \beta^{[j_2]}, \ldots, \beta^{[j_k]}$ represent conjugacy classes of $\beta^{j_1}, \beta^{j_2m}, \ldots \beta^{j_k}$ respectively with the corresponding minimal polynomials $y_{j_1}, y_{j_2}, \ldots, y_{j_k}$. For any codeword vector $\mathbf{c} \in \mathscr{C}$, the elements $\{c_{e_j}, c_{e_j+1}, \ldots, c_{n-1}\}$ are linear combinations of the first e_j elements $\{c_0, c_1, \ldots, c_{e_j-1}\}$, with e_j being the degree of the polynomial $y_j = lcm(y_{j_1}, y_{j_2}, \ldots, y_{j_k})$.*

Proof. Refer, Appendix A.1.

Proposition 1 implies that for any vector $\mathbf{c} \in \mathscr{C}$, $n-e_j$ elements of $\mathbf{c}$, starting from index e_j, are dependent on first e_j elements (linear combination of previous first e_j elements). In terms of codeword matrix $\mathbf{C}$, this implies that the $n - e_j$ columns starting from index e_j are dependent on the first e_j columns. This means that the rank of any codeword vector $\mathbf{c}$ depends on the first e_j elements of the vector. We make use of this result to prove the following Propositions.

Proposition 2. *Let $\mathscr{C}$ be a cyclic code over $\mathbb{F}_{q^m}$ designed using free transform components $U_J = \{U_{j_1}, U_{j_2}, \ldots, U_{j_k}\}$, with indices $J = \{j_1, j_2, \ldots, j_k\}$ and $j_1 \neq j_2 \neq \cdots \neq j_k$ (The other transform domain components are explicitly set to zero)*

Case 1: If $j_1, j_2, \ldots, j_{n-1} \in [j]_n$, then

$$R_q(\mathscr{C}) = r_j - (k-1)g$$

Where, r_j is the size of $q-$cyclotomic coset $[j]_n$, and $g|\mathscr{G}_1, g|\mathscr{G}_2, \ldots, g|\mathscr{G}_k, g|m$ such that: $\mathbb{F}_{q^g}|\mathbb{F}_{q^{\mathscr{G}_1}}, \mathbb{F}_{q^g}|\mathbb{F}_{q^{\mathscr{G}_2}}$ and so on Here, $\mathscr{G}_i = gcd(s_i, r_j)$, $1 \leq i \leq k$.

Case 2: If $j_1 \in [j_1]_n, j_2 \in [j_2]_n, \ldots, j_{n-1} \in [j]_n$, then

$$R_q(\mathscr{C}) = min(r_{min}, m)$$

Where, $r_{min} = min(r_{j_1}, r_{j_2}, \ldots, r_{j_{n-1}})$ and r_{j_1} is the size of $q-$ cyclotomic coset $[j_1]_n$, r_{j_2} is the size of $q-$ cyclotomic coset $[j_2]_n$ and so on.

Proof. Refer Appendix A.2.

From Case 2 of Proposition 2, one can see that full rank codes over $\mathbb{F}_{q^m}$ can be obtained by choosing free-transform components such that the index of each free-transform domain component comes from a different $q-$cyclotomic coset of size m.

Following Propositions 1 and 2, we see that the value of $e_j = km$ for any full rank codes.

3.2.3. Full rank codes

From case 2 of Proposition 2, we see that the full rank codes over $\mathbb{F}_{q^m}$ can be obtained by choosing free-transform components such that, the index of each free-transform domain component comes from a different $q-$cyclotomic coset of size m. Additionally, following Proposition 1, we see that the rank of these codes is dependent on first $e_j = km$ columns. Hence, (n, k) full rank codes can be punctured to (e_j, k) full rank codes, without an affect to the rank. In other words, $1 \times n$ full rank codeword vectors that are obtained using Proposition 2 can be punctured to yield

Table 2
Gaussian-Integers fields $\mathcal{G}_\pi$ for various values of π.

$q(\pi)$	$\mathcal{G}_\pi$
5 $(2+i)$	$\{0, 1, i, -1, -i\}$
13 $(3+2i)$	$\{0, 1, 1+i, 2i, -i, 1-i, 2, -1, -1-i, -2i, i, -1+i, -2\}$
17 $(4+i)$	$\left\{\begin{matrix} 0, 1, 1+i, 2i, -1-2i, i, -1+i, -2, 2-i, \\ -1, -1-i, -2i, 1+2i, -i, 1-i, 2, -2+i \end{matrix}\right\}$

Table 3
Eisenstein–Jacobi Integers fields $\mathcal{J}_\Pi$ for various values of Π.

$q(\Pi)$	$\mathcal{J}_\Pi$
$7(3+\rho\cdot 2)$	$\{0, 1, 1+\rho, \rho, -1, -1-\rho, -\rho\}$
$13(3+\rho\cdot 4)$	$\left\{\begin{matrix} 0, 1, 1+2\rho, 1+\rho, -1+\rho, \rho, -2-\rho, \\ -1, -1-2\rho, -1-\rho, 1-\rho, -\rho, 2+\rho, \end{matrix}\right\}$

a full rank codeword vector of size $1 \times e_j$. Equivalently, the full rank codeword matrices $\mathbf{C}$ of dimension $m \times n$ can be reduced to codeword matrices of dimension $m \times e_j$. The punctured codeword matrix $\mathbf{C}_R$ of $\mathbf{C}$ is given as,

$$\mathbf{C}_R = \begin{bmatrix} c_{0,0} & c_{0,1} & c_{0,2} & \cdots & c_{0,e_j-1} \\ c_{1,0} & c_{1,1} & c_{1,2} & \cdots & c_{1,e_j-1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ c_{m-1,0} & c_{m-1,1} & c_{m-1,2} & \cdots & c_{m-1,e_j-1} \end{bmatrix}$$

Where, $m \le e_j < n$, $0 \le c_{i,j} \le q-1$, $0 \le i \le m-1$, $0 \le j \le n-1$. Hence, $\mathscr{C}_R$ forms an (km, k) full rank punctured code of $\mathscr{C}$. The punctured full rank codeword matrices are used as NSFBCs for MIMO-OFDM systems with index modulation for the next section.

(a) Full rank codes as NSFBCs

The $m \times e_j$ codeword matrices of full rank codes $\mathscr{C}$ can be used in $N_T \times N_R$ MIMO systems with $N_T = m$ antennas. However, the codeword matrices that are obtained cannot be directly used in MIMO communication due to the presence of error correcting structure. Hence, they result in non-full rate (rate$< N_T$) integer codeword matrices [18]. A rank-preserving map is used to map symbols of codeword matrices $\mathbf{C}$ between symbols from $\mathbb{F}_q$ and complex constellations [19]. In [13], the map between Galois field $\mathbb{F}_q$ with $q = 4\mathscr{K} + 1$, $\mathscr{K} \ge 0$ and Gaussian integer field with $q = \mathfrak{u} + i\mathfrak{v}$ is shown to be rank preserving [20]. Where, $\mathfrak{u}$ and $\mathfrak{v}$ are integers. This map is used to construct STBCs from Gabidulin codes. Based on the analysis given in [13], Puchinger et al. [21] showed that for $q = 6\mathscr{K} + 1$, $\mathscr{K} \ge 0$ the map between Finite field $\mathbb{F}_q$ and Eisenstein–Jacobi integer field with $q = \mu + \rho\nu$, $\mu, \nu \ne 0$, $\rho = (-1 + i\sqrt{3})/2$ is rank preserving [22] and are used to construct STBCs. Tables 2 and 3 gives the Gaussian and Eisenstein–Jacobi integer fields that are used in this paper.

Space–frequency block codes (SFBC) can be obtained from codewords $\mathbf{C}$ of full rank code $\mathscr{C}$ by mapping each symbol of the codeword matrix $\mathbf{C}$ into symbol of Gaussian integer constellation or Eisenstein–Jacobi integer constellation. The resulting SFBC is shown below.

$$\mathbf{X}_\rho = \begin{bmatrix} \zeta(c_{00}) & \zeta(c_{01}) & \zeta(c_{02}) & \cdots & \zeta(c_{0\,e_j-1}) \\ \zeta(c_{10}) & \zeta(c_{11}) & \zeta(c_{12}) & \cdots & \zeta(c_{1\,e_j-1}) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \zeta(c_{m-10}) & \zeta(c_{m-11}) & \zeta(c_{m-12}) & \cdots & \zeta(c_{m-1\,e_j-1}) \end{bmatrix}$$

Where, ζ is either Gaussian integer map or Eisenstein–Jacobi integer map based on the value q of the field $\mathbb{F}_q$. The obtained SFBC codewords have columns that are generally having non-orthogonal nature. Hence, the resultant SFBC can be termed as non-orthogonal SFBC (NSFBC).

(b) NSFBC codebook formulation

We have seen that the full rank codes over $\mathbb{F}_{q^m}$ can be constructed by choosing k free transform components, with each index chosen from a different $q-$cyclotomic coset of size m. If there are $\mathcal{L} \ge k$ $q-$ cyclotomic cosets, each of which are having size m, then we can have $\binom{\mathcal{L}}{k}$ possible choices of k $q-$ cyclotomic cosets that are grouped together. Of which, k free transform component indices can be chosen at a time with each from different $q-$ cyclotomic coset. Since the $q-$ cyclotomic cosets are of same size m, there are m^k possible choices of $k-$ free transform components. Each choice produces a full rank cyclic code $\mathscr{C}_i$ with q^{km} codewords (matrices).

It can be stated that the rank distance between any two codewords (rank of difference matrix) of different codes such as $\mathscr{C}_i$ *and* $\mathscr{C}_j$ $0 \le i, j \le \binom{\mathcal{L}}{k} m^{k-1}$, $i \ne j$ may be less than m. The difference matrix which is obtained may not be a codeword and due to this, it is clearly understood that the rank goes down. However, from Case 2 of Proposition 2, one can see that there exists a few codes whose codewords can be at a rank-distance m with respect to the codewords within the code and a few other codes as well (following Case 2 of Proposition 2). The reason here is that no two indices are chosen from the same $q-$cyclotomic coset. Thus, the codewords that belong to these two component codes of rank m can be grouped to form a composite code $\mathscr{X}$ having rank m. The composite code can be represented by $\mathscr{X} = \cup_{i=0}^{m^{\mathscr{L}}-1} \mathscr{C}_i$. A composite NSFBC code $\mathscr{X}_{NSFBC}$ corresponding to composite NSFBC code $\mathscr{X}$ has been obtained using rank preserving maps. Since all zero codeword is common for all component codes, the number of codewords in a composite NSFBC code $\mathscr{X}_{NSFBC}$ will be $\left(\binom{\mathcal{L}}{k} q^{km} - 1\right) + 1$.

The working of MIMO-OFDM-IM system employing the proposed composite NSFBCs is detailed in the next subsection.

3.3. Working principle of NSFBC-MIMO-OFDM system

Following the general case given in Section 3.1, we propose the NSFBC based MIMO-OFDM-IM system with $N_F = GN$–OFDM carriers. Fig. 3 illustrates the working of proposed NSFBC based MIMO-OFDM-IM system employing $N_F > N$ subcarriers. To facilitate the use of NSFBCs, we consider the MIMO-OFDM system with $N_T = m$–number of transmit antennas and N_R–number of receive antennas. Since there are $N_F = GN$ number of subcarriers that are available for communication, we use G–NSFBC-IM blocks at the transmitter, with each block providing $N_T \times N$ NSFBC-IM codeword. Thus, the available N_F carriers are divided among these G blocks, with $N = N_F/G$ subcarriers per block. The value of G is selected in such a way that the adjacent subcarriers are uncorrelated. i.e. B_c/G is less than subcarrier spacing.

3.3.1. Transmitter

In Section 3.2.3, we have seen that there are $\binom{\mathcal{L}}{k}\left(q^{km} - 1\right) + 1$ codewords per $\mathscr{X}_{NSFBC}$. Since $N_T = m$, $m_b G = \left(log_2\left(\binom{\mathcal{L}}{k}(q^{kN_T} - 1) + 1\right) + n_m \cdot log_2\binom{N}{e_j}\right)$ G–number of input bits are considered at the input of splitter block. These input bits are then split into G groups where, each group consisting of $m_b = log_2\left(\binom{\mathcal{L}}{k}(q^{kN_T} - 1) + 1\right) + n_m \cdot log_2\binom{N}{e_j}$–number of information bits (Ib). At each NSFBC-IM block, the m_b information bits are further split into $p_1 = log_2\left(\binom{\mathcal{L}}{k}\left(q^{kN_T} - 1\right) + 1\right)$–number of data bits (Db); and $p_2 = n_m \cdot log_2\binom{N}{e_j}$–number of carrier selection bits (Csb). The p_1 data bits are processed by the NSFBC encoder to obtain the corresponding NSFBC codeword. The NSFBC codeword $\mathbf{X}_\rho$ is then considered as input for the index modulator (IM). Based on the carrier selection bits p_2, the index modulator select $N_A = e_j$ carriers out of available N subcarriers. The subcarriers that are chosen can be

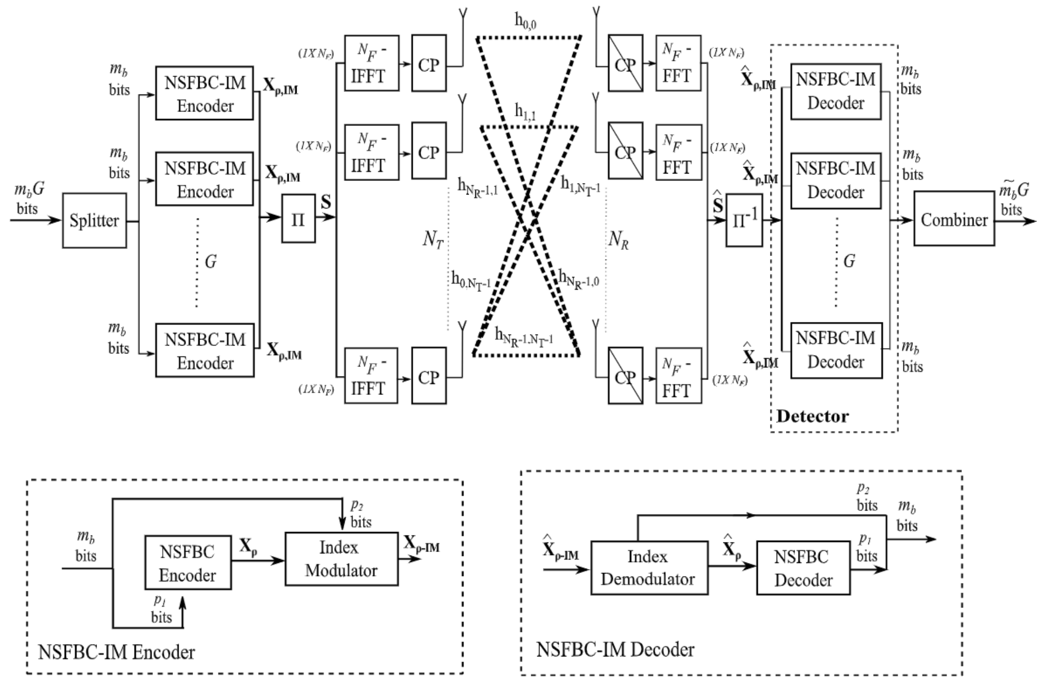

Fig. 3. Block-Diagram of NSFBC based $N_T \times N_R$ MIMO-OFDM-IM system.

(**1**) Same for all the rows of NSFBC (full rank mapping).
(**2**) Different for each row of NSFBC (rank-deficient mapping).

(**1**) In the first case, at any instance of time, the subcarriers that are chosen for all the rows of NSFBC codeword X_ρ are same. This process is to preserve the full rank property. Hence, $p_2 = log_2\binom{N}{e_j}$. This process give rise to $N_T \times N$–NSFBC-IM codewords, with all symbols along each column being either zero (if the carrier frequency is not chosen) or non-zero (if the carrier frequency is chosen). An example representation of NSFBC-IM codeword $\mathbf{X}_{\rho-IM}$ is given as.

$$\mathbf{X}_{\rho,IM} = \begin{bmatrix} X_{0,0} & X_{0,1} & \cdots & X_{0,N-1} \\ X_{1,0} & X_{1,1} & \cdots & X_{1,N-1} \\ \vdots & \vdots & \cdots & \vdots \\ X_{N_T-1,0} & X_{N_T-1,1} & \cdots & X_{N_T-1,N-1} \end{bmatrix} = \begin{bmatrix} \zeta(c_{0,0}) & 0 & \cdots & \zeta(c_{0,e_j-1}) \\ \zeta(c_{1,0}) & 0 & \cdots & \zeta(c_{1,e_j-1}) \\ \vdots & \vdots & \cdots & \vdots \\ \zeta(c_{N_T-1,0}) & 0 & \cdots & \zeta(c_{N_T-1,e_j-1}) \end{bmatrix} \quad (4)$$

Thus, the symbols along each column of $\mathbf{X}_{\rho,IM}$ are either 0 or $\zeta(c_{k,l})$, with $0 \leq k \leq m-1$, $0 \leq l \leq e_j - 1$. The codewords obtained are termed as full rank NSFBC-IM codewords and corresponding codes as full rank NSFBC-IM codes (FR NSFBC-IM).

(**2**) In the second case, the index modulator selects different subcarriers for different rows of NSFBC codeword. Hence, in this case $p_2 = mlog_2\binom{N}{e_j}$. This process gives rise to $N_T \times N$ NSFBC-IM codewords, with each symbol along each row being either zero (if the carrier frequency is not chosen) or non-zero (if the carrier frequency is chosen). An example representation of NSFBC-IM codeword $\mathbf{X}_{\rho-IM}$ is given below:

$$\mathbf{X}_{\rho,IM} = \begin{bmatrix} X_{0,0} & X_{0,1} & \cdots & X_{0,N-1} \\ X_{1,0} & X_{1,1} & \cdots & X_{1,N-1} \\ \vdots & \vdots & \cdots & \vdots \\ X_{N_T-1,0} & X_{N_T-1,1} & \cdots & X_{N_T-1,N-1} \end{bmatrix} = \begin{bmatrix} \zeta(c_{0,0}) & \zeta(c_{0,1}) & \cdots & 0 \\ 0 & \zeta(c_{1,0}) & \cdots & \zeta(c_{1,e_j-1}) \\ \vdots & \vdots & \cdots & \vdots \\ \zeta(c_{N_T-1,0}) & \zeta(c_{N_T-1,1}) & \cdots & 0 \end{bmatrix} \quad (5)$$

From (5), based on the carrier selection bits, it is observed that the element $X_{i,j}$ ($0 \leq i \leq N_T - 1$, $0 \leq j \leq N-1$) of $\mathbf{X}_{\rho,IM}$ is either $\zeta(c_{k,l})$ $\left(0 \leq k \leq N_T - 1,\ 0 \leq l \leq e_j - 1\right)$ or 0. Hence, the codewords obtained are termed as rank deficient NSFBC-IM codewords and corresponding codes are termed rank deficient NSFBC-IM codes (RD NSFBC-IM).

In this work, we have considered FR-NSFBC-IMs for MIMO-OFDM-IM system with $N_R < 4$ antennas and RD-NSFBC-IMs for MIMO-OFDM-IM system with $N_R \geq 4$ antennas. In any case, the output of index modulator, $\mathbf{X}_{\rho,IM}$, is given by either (4) or (5). For convenience, let $\mathbf{X}_{\rho,IM}$ be represented as

$$\mathbf{X}_{\rho,IM} = \{\boldsymbol{X}_0, \boldsymbol{X}_1, \boldsymbol{X}_2 \ldots, \boldsymbol{X}_{N-1}\},$$

where, $\boldsymbol{X}_\rho$ is the ρth column of the NSFBC-IM codeword $\mathbf{X}_{\rho,IM}$

At the transmitter, each NSFBC-IM block gives rise to one NSFBC-IM codeword. Since there are G NSFBC-IM blocks, G NSFBC-IM codewords are available at the input of interleaver (Π). The interleaver stacks $G = N_F/N$ such NSFBC-IM codewords to form one NSFBC-OFDM-IM block $\mathbf{S}$, given by

$$\begin{aligned} \mathbf{S} &= \left\{\mathbf{X}^0_{\rho,IM}, \mathbf{X}^1_{\rho,IM}, \ldots, \mathbf{X}^{G-1}_{\rho,IM}\right\} \\ &= \left\{\boldsymbol{X}^0_0, \boldsymbol{X}^0_1, \ldots, \boldsymbol{X}^0_{N-1}, \boldsymbol{X}^1_0, \boldsymbol{X}^1_1, \ldots, \right. \\ &\quad \left. \boldsymbol{X}^1_{N-1}, \boldsymbol{X}^{G-1}_0, \boldsymbol{X}^{G-1}_1, \ldots, \boldsymbol{X}^{G-1}_{N-1}\right\}. \end{aligned} \quad (6)$$

Where, $\mathbf{X}^{\mathcal{G}}_{\rho,IM}$ is the $\mathcal{G}$th, $0 \le \mathcal{G} \le G-1$, NSFBC-IM codeword of $\mathbf{S}$. $\boldsymbol{X}^{\mathcal{G}}_{\rho}$ is the ρth column of the $\mathcal{G}$th NSFBC-IM codeword $\mathbf{X}^{i}_{\rho,IM}$, given by

$$\boldsymbol{X}^{\mathcal{G}}_{\rho} = \left[X^{\mathcal{G}}_{0,\rho}, X^{\mathcal{G}}_{1,\rho}, X^{\mathcal{G}}_{2,\rho} \dots, X^{\mathcal{G}}_{N_T-1,\rho}\right]^T; \quad 0 \le \rho \le N-1, 0 \le \mathcal{G} \le G-1$$

The terms $\mathcal{G}$ and ρ can be related as $\mathcal{G} = \lfloor \frac{\rho}{N} \rfloor$. Here, $G = N_F/N$, is number of NSFBC-IM codewords with N carriers per NSFBC-IM codeword.

Since the interleaving is in the frequency dimension, the number of rows of $\mathbf{S}$ remain same as that of $\mathbf{X}_{\rho,IM}$, i.e. equal to N_T. Each row of the NSFBC-IM-OFDM block $\mathbf{S}$ is then sent to the corresponding IFFT block. Further, the IFFT block computes $1 \times N_F$ time domain vector. The obtained N_F-point time domain vector is then padded with a cyclic prefix (CP) and then modulated onto $N_F + N_{CP}$ carriers. Since, the IFFT computation occurs at each transmit antenna (corresponding to each row of $\mathbf{S}$), N_T–IFFT vectors are available at the output of transmitter. These N_T–IFFT vectors are transmitted simultaneously over MIMO channel. For better resilience to inter-symbol interference (ISI), the length of cyclic prefix L, is chosen to be greater than the channel length L.

Spectral efficiency

The spectral efficiency is defined as the number of bits transmitted per channel usage per carrier frequency and is given by

$$\eta = \frac{G.\left(log_2\left(\binom{\mathcal{L}}{k}(q^{kN_T}-1)+1\right) + n_m \cdot log_2\binom{N}{e_j}\right)}{N_F + N_{CP}},$$

$$n_m = \begin{Bmatrix} 1 & FR-NSFBC-IM \\ N_T & RD-NSFBC-IM \end{Bmatrix}$$

3.3.2. Channel

The channel is considered to be frequency selective and time-flat with $L <$ L (length of cyclic prefix)–number of taps. Fading between each transmitting and receiving antenna is considered to be independent and identically distributed (I.I.D) with the Rayleigh distribution at a particular carrier frequency ρ. The channel coefficients among various subcarriers are also considered to be identically distributed. Further assumption has been made that the wireless channels remain constant during the transmission of a MIMO-OFDM-IM frame.

3.3.3. Receiver

After removing the CP of length L per OFDM symbol and applying FFT at each branch of the receiver, the received $N_R \times N_F$ matrix at the input of deinterleaver is given by,

$$\begin{aligned}\mathbf{Y} &= \left\{\mathbf{Y}^0, \mathbf{Y}^1, \dots, \mathbf{Y}^{\mathcal{G}-1}\right\} \\ &= \left\{\boldsymbol{Y}^0_0, \boldsymbol{Y}^0_1, \dots, \boldsymbol{Y}^0_{N-1}, \boldsymbol{Y}^1_0, \boldsymbol{Y}^1_1, \dots, \boldsymbol{Y}^{G-1}_1, \boldsymbol{Y}^{G-1}_0, \boldsymbol{Y}^{G-1}_1, \dots, \boldsymbol{Y}^{G-1}_{N-1}\right\}\end{aligned}$$

with each column $\boldsymbol{Y}^{\mathcal{G}}_{\rho}$ representing the $N_R \times 1$ received vector at particular frequency ρ (after FFT), given by

$$\boldsymbol{Y}^{\mathcal{G}}_{\rho} = \begin{bmatrix} y^{\rho}_0 \\ y^{\rho}_1 \\ y^{\rho}_2 \\ \vdots \\ y^{\rho}_{N_R-1} \end{bmatrix} = \begin{bmatrix} H^{\rho}_{0,0} & H^{\rho}_{0,1} & \cdots & H^{\rho}_{0,N_T-1} \\ H^{\rho}_{1,0} & H^{\rho}_{1,1} & \cdots & H^{\rho}_{1,N_T-1} \\ H^{\rho}_{2,0} & H^{\rho}_{2,1} & \cdots & H^{\rho}_{2,N_T-1} \\ \vdots & \vdots & \vdots & \vdots \\ H^{\rho}_{N_R-1,0} & H^{\rho}_{N_R-1,1} & \cdots & H^{\rho}_{N_R-1,N_T-1} \end{bmatrix} \begin{bmatrix} X^{\mathcal{G}}_{0,\rho} \\ X^{\mathcal{G}}_{1,\rho} \\ X^{\mathcal{G}}_{2,\rho} \\ \vdots \\ X^{\mathcal{G}}_{N_T-1,\rho} \end{bmatrix} + \begin{bmatrix} w^{\rho}_0 \\ w^{\rho}_1 \\ w^{\rho}_2 \\ \vdots \\ w^{\rho}_{N_R-1} \end{bmatrix}$$

$$0 \le \rho \le N_F - 1 \tag{7}$$

Where, $H^{\rho}_{\omega\Omega} = \sum_{l=0}^{L-1} h_{\omega\Omega}(l) e^{-j\left(\frac{2\pi}{N_c}\right)l\rho}$; with $\mathbf{h}_{\omega\Omega} \equiv [h_{\omega\Omega}(0), h_{\omega\Omega}(1), \dots, h_{\omega\Omega}(L-1)]$ representing the baseband equivalent impulse response of the channel between ωth transmit antenna and Ωth receive antenna, and L denoting the length of the channel impulse response. Following (6), for simplicity, (7) can be written as

$$\boldsymbol{Y}^{\mathcal{G}}_{\rho} = \mathbf{H}^{\mathcal{G}}_{\rho} \boldsymbol{X}^{\mathcal{G}}_{\rho} + \boldsymbol{W}^{\mathcal{G}}_{\rho}; \qquad 0 \le \rho \le N_F - 1 \ \& \ \mathcal{G} = \left\lfloor \frac{\rho}{N} \right\rfloor \tag{8}$$

Where, $\boldsymbol{W}^{\mathcal{G}}_{\rho}$ is the $N_R \times 1$ vector of elements that are realizations of Gaussian random variable with zero mean. $\mathbf{H}^{\mathcal{G}}_{\rho}$ is $N_R \times N_T$ baseband equivalent impulse response of the channel matrix at a particular frequency ρ.

The deinterleaver (Π^{-1}) considers the received matrix, constructs a block of $N-$ columns, and feeds the $N_R \times N$ matrix to the detector. The detection employed is either a single-stage Maximum Likelihood (ML) detection of the entire NSFBC-IM code, or a two-stage minimum mean square estimation (MMSE)-ML decoder. In the case of two-stage MMSE-ML decoder, the first stage is for the detection of carrier selection bits using Minimum Mean Square Estimation (MMSE), and the second stage is for the detection of data bits using Maximum Likelihood (ML) decoder. Thus the receiver is either

1. Single stage ML receiver or
2. Two Stage MMSE-ML receiver

1 Single Stage ML Receiver

In this case, the NSFBC-IM decoder is considered to be ML detector. At particular subblock G, the ML detector considers the $N_R \times N$ matrix $\mathbf{Y}^{\mathcal{G}}$ corresponding to that subblock G and obtains an estimate of the transmitted $N_T \times N$ matrix for estimating the transmitted NSFBC-IM codeword $\mathbf{X}_{\rho-IM}$. Following (7), the received $N_R \times N$ matrix can be given as

$$\mathbf{Y}^{\mathcal{G}} = \hat{\mathbf{H}}^{\mathcal{G}} \left(\mathbf{X}^{\mathcal{G}}_{\rho,IM}\right)^T + \mathbf{W}^{\mathcal{G}}; \qquad 0 \le \mathcal{G} \le G-1 \tag{9}$$

Where,

$$\mathbf{Y}^{\mathcal{G}} = \left[\boldsymbol{Y}^{\mathcal{G}}_0, \boldsymbol{Y}^{\mathcal{G}}_1, \dots, \boldsymbol{Y}^{\mathcal{G}}_{N-1}\right]^T;$$

$$\hat{\mathbf{H}}^{\mathcal{G}} = \begin{bmatrix} \hat{\mathbf{H}}^{\mathcal{G}}_0 & 0 & 0 & \cdots & 0 \\ 0 & \hat{\mathbf{H}}^{\mathcal{G}}_1 & 0 & \cdots & 0 \\ 0 & 0 & \hat{\mathbf{H}}^{\mathcal{G}}_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \hat{\mathbf{H}}^{\mathcal{G}}_{N-1} \end{bmatrix},$$

$$\mathbf{X}^{\mathcal{G}}_{\rho,IM} = \left[\mathbf{X}^{\mathcal{G}}_0, \boldsymbol{X}^{\mathcal{G}}_1, \mathbf{X}^{\mathcal{G}}_2, \dots, \mathbf{X}^{\mathcal{G}}_{N-1}\right];$$

$$\mathbf{W}^{\mathcal{G}} = \left[\boldsymbol{W}^{\mathcal{G}}_0, \boldsymbol{W}^{\mathcal{G}}_1, \dots, \boldsymbol{W}^{\mathcal{G}}_{N-1}\right]$$

The ML decision required in estimating one transmitted NSFBC-IM codeword $\mathbf{X}^{\mathcal{G}}_{\rho,IM}$, is then given by

$$\left(\mathbf{X}^{\mathcal{G}}_{\rho,IM}\right)_{ML} = \arg\min\left(\left\|\mathbf{Y}^{\mathcal{G}} - \hat{\mathbf{H}}^{\mathcal{G}}\left(\mathbf{X}^{\mathcal{G}}_{\rho,IM}\right)^T\right\|^2_F\right); \quad 0 \le \mathcal{G} \le G-1 \tag{10}$$

From the estimated NSFBC-IM matrix, the information bits (Ib) transmitted are then decoded.

2 MMSE-ML Based Receiver

(a) In this case the detection is a two-stage process. At the first stage, the information conveyed using index modulator is obtained using MMSE based index demodulator. At the second stage, the modulated data is obtained using ML based NSFBC decoder. Assuming availability of perfect channel state information

Table 4
Computational complexity.

Method	Complexity
MIMO-OFDM	$N_R(N_T+1)M^{N_T}$
MIMO-OFDM-IM	$N_R(N_T+1)(2^{p_1}M^K)^{N_T}$
Proposed Method (MMSE-ML)	$N_RN_T\,(2N_R+N)+e_jm\left(\binom{\mathcal{L}}{k}\left(q^{km}-1\right)+1\right)$
Proposed Method (ML)	$\binom{N}{e_j}^m\left(\binom{\mathcal{L}}{k}(q^{km}-1)+1\right)\left(N_RN_TN^2+N_TN\right)$

(CSI) at the receiver the MMSE estimation of $\hat{\mathbf{X}}_\rho^{\mathcal{G}}$ can be obtained in the following manner. At the first stage, MMSE estimation is performed for each column of the decoded NSFBC-OFDM-IM block $\mathbf{S}$. The estimated vector $\hat{\mathbf{X}}_\rho^{\mathcal{G}}$ is obtained using

$$\hat{\mathbf{X}}_\rho^{\mathcal{G}}=\mathbf{H}_\rho^{\mathcal{G}H}\left(\mathbf{H}_\rho^{\mathcal{G}}\mathbf{H}_\rho^{\mathcal{G}H}+\lambda\mathbf{I}\right)^{-1}\mathbf{Y}_\rho^{\mathcal{G}}\ ;\quad 0\le\rho\le N_F-1$$

$$\hat{\mathbf{X}}_\rho^{\mathcal{G}}=\mathbf{H}_\rho^{\mathcal{G}H}\left(\mathbf{H}_p^{\mathcal{G}}\mathbf{H}_\rho^{\mathcal{G}H}+\lambda\mathbf{I}\right)^{-1}\left(\mathbf{H}_\rho^{\mathcal{G}}\boldsymbol{X}_\rho^{\mathcal{G}}+\boldsymbol{W}^{\rho}\right)\qquad(11)$$

Where, λ is a constant and is equal to $\frac{N_T}{(\frac{E_b}{N_0})}$ in the case of linear MMSE estimators.
An estimate of the transmitted NSFBC-IM is then given by

$$\hat{\mathbf{X}}_{\rho,IM}^{\mathcal{G}}=\left\{\hat{\mathbf{X}}_0^{\mathcal{G}},\hat{\mathbf{X}}_1^{\mathcal{G}},\hat{\mathbf{X}}_2^{\mathcal{G}},\ldots,\hat{\mathbf{X}}_{N-1}^{\mathcal{G}}\right\}$$

As seen from (11), $\hat{\mathbf{X}}_{\rho,IM}^{\mathcal{G}}$ contains Gaussian noise which is scaled by fading coefficients. From $\hat{\mathbf{X}}_{\rho,IM}^{\mathcal{G}}$, it is possible to obtain the carrier selection bits by finding locations of minimum magnitude. The remaining e_j high magnitude complex values of each row are considered to form an estimate of the $m\times e_j$ NSFBC matrix. This matrix is then fed as input to NSFBC decoder.

(b) The second decoder (NSFBC decoder) considers the estimated $m\times e_j$ matrix and compares it with all the codewords of the corresponding composite NSFBC $\mathcal{X}_i$. The codeword which is at the minimum distance with this $m\times e_j$ matrix is then considered as $\hat{\mathbf{X}}_p$. The information corresponding to $\hat{\mathbf{X}}_p$ is considered as data bits.

The carrier selection and data bits together form an estimate of the transmitted information bits.

3.3.4. Computational complexity

In this Section, The computational complexity (in terms of the number of complex multiplications) of the receiver in decoding information which is pertaining to one NSFBC codeword is discussed. The terms K, p_1, M mentioned in Table 4 are taken from [4]. Where, K represents the number of subcarriers assigned per MIMO-OFDM-IM codeword, M is the order of QAM used, and p_1 is the number of data bits per group in MIMO-OFDM-IM system.

As seen from Table 4, the single stage ML decoding complexity of NSFBC-MIMO-OFDM-IM is exponential in $\binom{N}{e_j}$ which is increasing the decoding complexity with m. However, the two-stage decoding complexity is found to be dependent only on q^{km}.

4. Analytical upper bound

In [6], an upper bound on the probability of error of Alamouti based space–frequency block codes (SFBC) is provided. The bound is derived by exploiting the structure of Alamouti code which is resulting in a closed form expression. Because of the frequency selective nature of the channel and structure of the Alamouti code, joint detection of the symbols has been employed [6]. However, in this approach, as the structure is non-orthogonal and also because of the assumption that L > L, we have employed the use of BER analysis provided in [4].

Since the value of G is considered in such a way that the subcarriers are uncorrelated, the pairwise error events within different subblocks are identical [4]. Hence, it is sufficient to estimate the PEP associated with one subblock to evaluate the performance of the proposed scheme. To derive the bound, we consider single stage ML detection of entire $N_T\times N$ NSFBC-IM codeword including joint detection of symbols along N_T rows of the received matrix, and for N consecutive columns that are corresponding to N columns of $\mathbf{X}_{\rho,IM}$.

Based on the analysis given in [4], the analytical upper bound (union bound) is derived with reference to (10) and is given as,

$$P_b\le\frac{1}{N_c}\sum_{i=0}^{N_c-1}\sum_{j=0}^{N_c-1}\frac{P\left(\mathbf{X}_{\rho,IM}^i\rightarrow\mathbf{X}_{\rho,IM}^j\right)n_{i,j}}{N_b};\qquad i\neq j\qquad(12)$$

Where,

- $N_c=\left(\binom{\mathcal{L}}{k}(q^{kN_T}-1)+1\right)\binom{N}{e_j}^{n_m}$—total number of NSFBC-IM codewords.
- N_b—total number of bits associated with the NSFBC-IM codeword.
- $n_{i,j}$—number of bits in error between binary tuple associated with $\mathbf{X}_{\rho,IM}^i$ and $\mathbf{X}_{\rho,IM}^j$.
- $P\left(\mathbf{X}_{\rho,IM}^i\rightarrow\mathbf{X}_{\rho,IM}^j\right)$ represents the Pairwise Error Probability (PEP).

In (12), PEP is obtained by computing conditional PEP (CPEP) between two NSFBC-IM codewords and averaging the CPEP over all channel realizations.

Following [23], the CPEP $P\left(\mathbf{X}_{\rho,IM}^i\rightarrow\mathbf{X}_{\rho,IM}^j\mid\hat{\mathbf{H}}_{\rho,IM}^{\mathcal{G}}\right)$ is given by,

$$P\left(\mathbf{X}_{\rho,IM}^i\rightarrow\mathbf{X}_{\rho,IM}^j\mid\hat{\mathbf{H}}_{\rho,IM}^{\mathcal{G}}\right)=Q\left(\sqrt{\frac{\rho\left\|\mathbf{H}_{\rho,IM}^{\mathcal{G}}\left(\mathbf{X}_{\rho,IM}^i-\mathbf{X}_{\rho,IM}^j\right)\right\|^2}{2}}\right)\qquad(13)$$

Using Craig's formula, this can be expressed as

$$P\left(\mathbf{X}_{\rho,IM}^i\rightarrow\mathbf{X}_{\rho,IM}^j\mid\hat{\mathbf{H}}_{\rho,IM}^{\mathcal{G}}\right)=\frac{1}{\pi}\int_0^{\pi/2}\exp\left(\sqrt{\frac{\rho\left\|\mathbf{H}_{\rho,IM}^{\mathcal{G}}\left(\mathbf{X}_{\rho,IM}^i-\mathbf{X}_{\rho,IM}^j\right)\right\|^2}{2}}\right)\qquad(14)$$

The Pairwise error probability (PEP) [4,23] can be obtained by integrating (14) over probability density of $\Gamma=\left\|\mathbf{H}_{\rho,IM}^{\mathcal{G}}\left(\mathbf{X}_{\rho,IM}^i-\mathbf{X}_{\rho,IM}^j\right)\right\|^2$. The PEP is given by

$$P\left(\mathbf{X}_{\rho,IM}^i\rightarrow\mathbf{X}_{\rho,IM}^j\right)=\frac{1}{\pi}\int_0^{\pi/2}\prod_{k=0}^{N_k}\left(\frac{1}{1+\frac{E_b\lambda_{i,j,k}}{4N_0\sin^2\phi}}\right)^{n_R}d\phi\qquad(15)$$

Where, N_k is the number of Eigen values, and $\lambda_{i,j,k}$ are the Eigen values of the difference matrix $\left(\mathbf{X}_{\rho,IM}^i\rightarrow\mathbf{X}_{\rho,IM}^j\right)\left(\mathbf{X}_{\rho,IM}^i\rightarrow\mathbf{X}_{\rho,IM}^j\right)^H$. It can be noted that, the value of $N_k=N_A=e_j$ in the case of FR-NSFBC-IM. Where as, the value of N_k can be less than e_j for RD-NSFBC-IM codewords.

Table 5
MIMO-OFDM-IM system parameters [4].

Number of subcarriers (N_F)	512
Number of subcarriers per NSFBC-IM Block (N)	4,8,16
Subcarrier spacing (Δf)	15 KHz
Sampling frequency (f_s)	7.68 MHz
Cyclic prefix length (L)	36
Gaussian or Eisenstein Constellations	$\mathbb{F}(5)$, $\mathbb{F}(7)$, $\mathbb{F}(13)$

Table 6
Spectrally efficiencies of proposed FR-NSFBC-IM codes over $\mathbb{F}_{q^2}$.

$\mathscr{C}_R-(e_j,k)$	N	$\eta_{theoretical}$ (bpcu)			$\eta_{practical}$ (bpcu)		
		q=5	q=7	q=13	q=5	q=7	q=13
(2, 1)	4	2.45	2.93	3.97	2.10	2.56	3.50
(4, 2)	8	2.44	2.92	3.79	2.33	2.80	3.73
(8, 4)	16	2.33	2.84	3.71	2.27	2.80	3.67

Table 7
Spectrally efficiencies of proposed FR-NSFBC-IM over $\mathbb{F}_{q^4}$.

$\mathscr{C}_R-(e_j,k)$	N	$\eta_{theoretical}$ (bpcu)			$\eta_{practical}$ (bpcu)		
		q=5	q=7	q=13	q=5	q=7	q=13
(2, 1)	4	2.64	3.10	3.93	2.56	3.03	3.85
(4, 2)	8	2.66	3.12	3.96	2.62	3.03	3.91
(8, 4)	16	2.64	3.10	3.94	2.62	3.09	3.91

Table 8
Spectrally efficiencies of proposed RD-NSFBC-IM codes over $\mathbb{F}_{q^4}$.

$\mathscr{C}_R-(e_j,k)$	N	$\eta_{theoretical}$ (bpcu)			$\eta_{practical}$ (bpcu)		
		q=5	q=7	q=13	q=5	q=7	q=13
(2, 1)	4	4.79	5.25	6.08	4.67	5.13	5.95
(4, 2)	8	5.05	5.51	6.35	4.90	5.31	6.18
(8, 4)	16	5.19	5.66	6.49	5.16	5.63	6.45

Following [23], using partial fraction expansion, a closed form solution of PEP can be given by,

$$P(\mathbf{X}^i_{\rho,IM} \to \mathbf{X}^j_{\rho,IM}) \leq \frac{1}{2}\prod_{l=1}^{N_k}\left(\frac{1}{(1+c_l)}\right)^{N_R} \tag{16}$$

Where, $c_l = \frac{E_b\lambda_{i,j,k}}{4N_0}$.

Substituting (16) in (12), union bound can be given by,

$$P_b \leq \frac{1}{N_c}\sum_{i=0}^{N_c-1}\sum_{j=0}^{N_c-1}\frac{\frac{1}{2}\prod_{l=1}^{N_k}\left(\frac{1}{(1+c_l)}\right)^{N_R} n_{i,j}}{N_b}; \quad i \neq j \tag{17}$$

$$P_b \leq \frac{1}{2N_cN_b}\sum_{i=0}^{N_c-1}\sum_{j=0}^{N_c-1}\prod_{l=1}^{N_k}\left(\frac{1}{(1+c_l)}\right)^{N_R} n_{i,j}; \quad i \neq j \tag{18}$$

5. Simulation results

Based on the criteria given in [4,12], the number of transmit antennas N_T is considered to be two and four. We consider a 10-path frequency-selective Rayleigh fading MIMO channel with the maximum delay spread of $10T_s$ for fair comparison with existing results [4]. Table 5 gives the values of various system parameters considered for simulations.

It is assumed that the channel state information (CSI) is unknown to the transmitter but, perfectly known to the receiver. Since, $N_F = 512$ and $N = 4,\ 8,\ or\ 16$, each value of N results in $G = G = N_F/N = 128,\ 64,\ or$ 32 number of NSFBC-IM codewords per NSFBC-OFDM-IM block.

Tables 6–8 give spectral efficiencies that can be achieved by the proposed codes for various (e_j, k) codes over $\mathbb{F}_{q^2}$ and $\mathbb{F}_{q^4}$ respectively. From Table 6, it can be seen that a theoretical spectral efficiency of around 2.45 b/s/Hz can be achieved by using codes over $\mathbb{F}_{5^2}$ when compared to 1.87 b/s/Hz offered by MIMO-OFDM-IM with BPSK [4]. From Table 7, we note that the improvement in spectral efficiency achieved by FR-NSFBC-IM codes over $\mathbb{F}_{q^4}$ is around 0.2 b/s/Hz for most of the values of q when compared to the corresponding FR-NSFBC-IM codes over $\mathbb{F}_{q^2}$. From Table 8, we see that the proposed RD-NSFBC-IM codes achieve better spectral efficiencies which is almost > 2 b/s/Hz while comparing with the corresponding FR-NSFBC-IM codes over $\mathbb{F}_{q^4}$. The reason is that the carrier selection bits per RD-NSFBC-IM codeword are n_m times more than FR-NSFBC-IM codeword.

In Fig. 4, the simulation results and upper bound of proposed (e_j, k) NSFBCs over $\mathbb{F}_{5^2}$, $\mathbb{F}_{7^2}$ are depicted. It can be seen that for a particular value of q, the BER performance improves with an increase in the values of e_j, k. It is observed that, (16, 8) codes over $\mathbb{F}_{5^2}$ provide a gain of around 1.5 dB when compared to (8, 4) codes over $\mathbb{F}_{5^2}$. The above results are observed at a BER of 10^{-5}. A similar pattern is also observed in the case of codes over $\mathbb{F}_{7^2}$. In addition to that, we see that the codes over $\mathbb{F}_{5^2}$ achieve a spectral efficiency of around 2.4 b/s/Hz from Table 6. Whereas, codes over $\mathbb{F}_{7^2}$ achieve a spectral efficiency of around codes over 2.9 b/s/Hz. From Fig. 4, it can also be seen that the proposed (8, 4) FR NSFBC-IM codes provide similar BER performance when compared to Rate-1 Alamouti code based MIMO-OFDM-IM. However, in a 2 × 2 MIMO-OFDM-IM system with $N_F = 512$ and $N = 4$, the Alamouti code with QPSK symbols (with rank preserving index modulation) provide a theoretical spectral efficiency of 1.5 b/s/Hz which is approximately 1 b/s/Hz less than the spectral efficiency provided by FR-NSFBC-IM codes over $\mathbb{F}_{5^2}$. Furthermore, it is observed that at lower values of (e_j, k), constructions over $\mathbb{F}_{7^2}$ provide similar performance with respect to constructions over $\mathbb{F}_{5^2}$. However, we see that the spectral efficiency is increased by about 0.3 b/s/Hz as shown in Table 6.

In Fig. 5, we compare the BER performance of 2 × 2 MIMO-OFDM-IM system employing NSFBC codes over $\mathbb{F}_{5^2}$, Rate-1 OSFBC (Alamouti) codes [6] and full rate quasi-orthogonal space frequency block codes (QOSFBC) [18]. The detection is based on single-stage ML decoding. Moreover, VBLAST based MIMO-OFDM scheme is also considered for comparison. One can observe from Fig. 5 that the proposed (8, 4) FR-NSFBC-IM codes provide an asymptotic gain of around 1 dB when compared to MIMO-OFDM-IM with BPSK uncoded constructions [4]. The improvement in spectral efficiency is about 0.6b/s/Hz. A similar asymptotic performance is observed with (8, 4) NSFBC code with an improved spectral efficiency of around 0.9 b/s/Hz when compared to Rate-1 Alamouti based MIMO-OFDM-IM (with rank preserving index mapping). It can be seen that (16, 8) NSFBC codes offer an asymptotic gain of around 0.5 dB when compared to QOSFBC [18] based MIMO-OFDM-IM system. The performance is approximately 2 dB when compared to OSFBC based MIMO-OFDM-IM system. In this case, the improvement in spectral efficiency when compared to OSFBC-MIMO-OFDM-IM and QOSFBC-MIMO-OFDM-IM is 0.5 and 0.9 b/s/Hz respectively. In case of a 4 × 4 MIMO system, the asymptotic performance of (8, 4) RD-NSFBC-IM codes over $\mathbb{F}_{5^4}$ is observed to be similar at a BER of 10^{-4}. Here, both simulation and analytical performances have been realized to offer a gain difference of approximately 0.1 dB when compared to the existing method [4]. However, from Table 8, the obtained spectral efficiency is found to be 1.3 b/s/Hz higher than the spectral efficiency quoted by MIMO-OFDM-IM system with BPSK [4]. Hence, the proposed RD-NSFBC-IM codes offer higher spectral efficiency, when compared with the uncoded MIMO-OFDM-IM scheme [4].

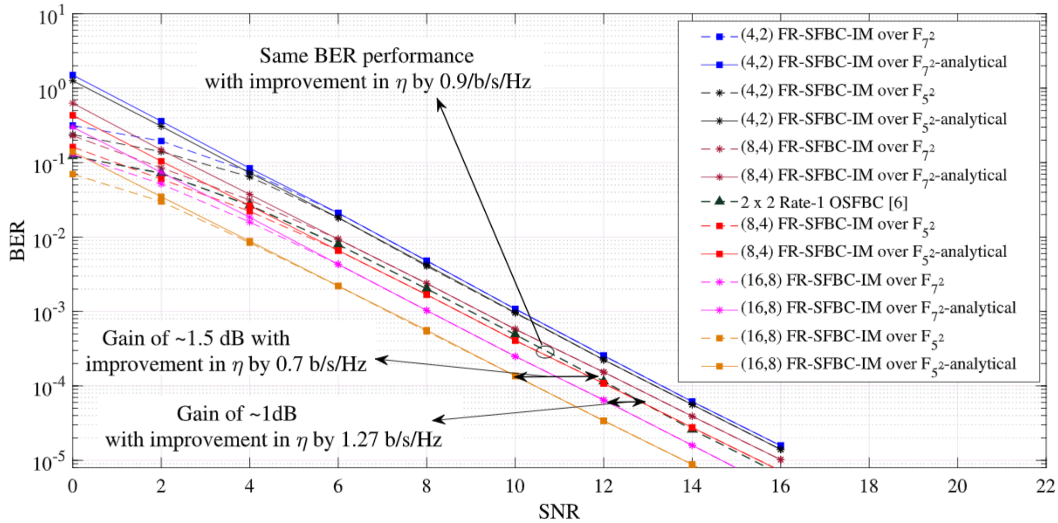

Fig. 4. BER performance of FR-NSFBC-IMs over $\mathbb{F}_{5^2}, \mathbb{F}_{7^2}$ with N_T=2, N_R=2, $N = 4, 8, 16$ corresponding to $e_j = 2, 4, 8$ and ML–ML decoding.

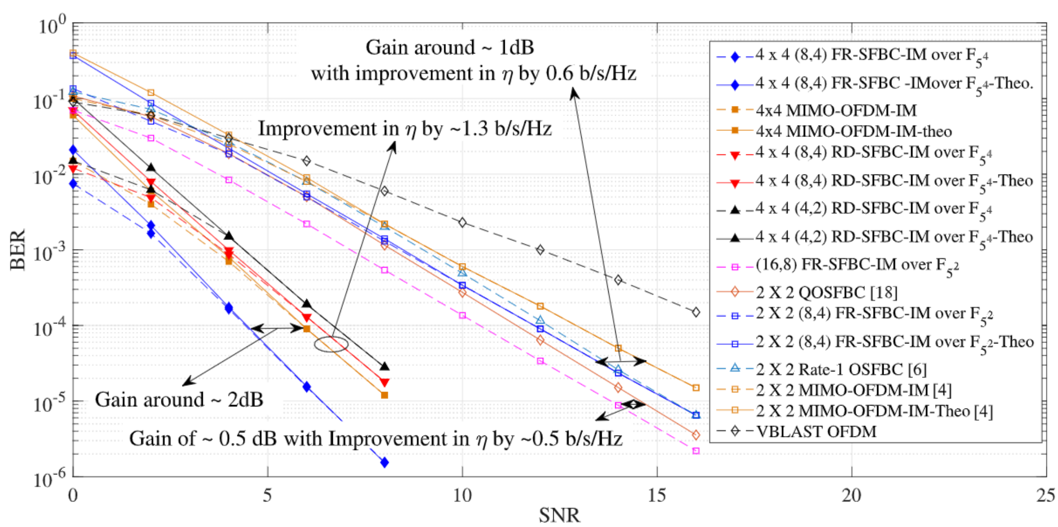

Fig. 5. BER performance of NSFBC-IMs (both FR and RD) over $\mathbb{F}_{5^2}$ and $\mathbb{F}_{5^4}$ for MIMO-OFDM-IM system with ML–ML decoding and $N_T = N_R$=2,4, N=4,8,16,32.

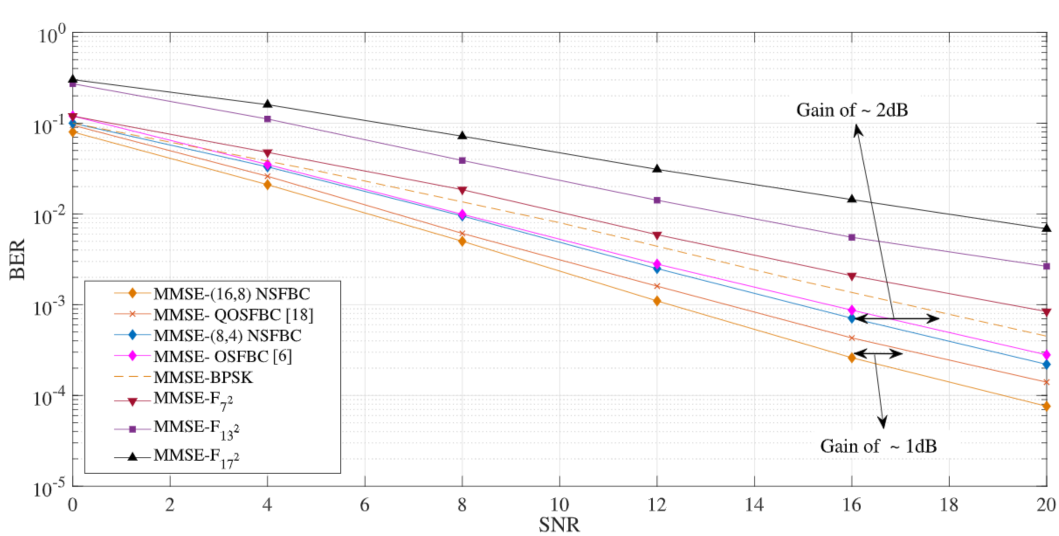

Fig. 6. BER of the proposed NSFBC over $\mathbb{F}_{5^2}, \mathbb{F}_{7^2}, \mathbb{F}_{13^2}, \mathbb{F}_{17^2}$, for MIMO-OFDM-IM system with MMSE-ML decoding and $N_T = N_R$=2, N=4 corresponding to e_j =8.

In this case, the BER performance is similar. In addition to that the proposed $(4, 2)$ outperforms VBLAST OFDM. However, both the systems maintain the same spectral efficiency.

In Fig. 6, the performance of 2×2 MIMO-OFDM-IM system employing MMSE detection has been depicted. We can see that the proposed constructions over $\mathbb{F}_{5^2}$ provide a gain of approximately 2 dB when compared to uncoded communication using BPSK [4]. This is observed when BER is 10^{-3}. In the case of $(8, 4)$ FR-NSFBC-IM codes over $\mathbb{F}_{13^2}$, $\mathbb{F}_{13^2}$, and $\mathbb{F}_{17^2}$, the BER performance gets deteriorated with an increase in the attribute q. However, Table 8 shows that the spectral efficiency of FR-NSFBC-IM codes over $\mathbb{F}_{7^2}$ is about 1.1 b/s/Hz higher than that of MIMO-OFDM-IM with BPSK [4]. It is also observed that $(16, 8)$ FR-NSFBC-IM code over $\mathbb{F}_{5^2}$ offer asymptotic gain of around 1 dB when compared to $(8, 4)$ FR-NSFBC-IM codes over $\mathbb{F}_{5^2}$. The spectral efficiency is same in both the cases.

6. Conclusion

In this paper, we have proved that there is a possibility to design full rank cyclic codes $k \geq 2$ and also detailed the process of constructing (n, k) full rank cyclic codes for $k \geq 2$. The full rank codes that are derived from cyclic codes have been employed to synthesize space–frequency block codes that are non-orthogonal in nature (NSFBCs) for MIMO-OFDM-IM systems. The performance of these systems has been evaluated on a 10-path frequency selective MIMO channel. The results obtained through simulation says that the proposed RD-NSFBC-IM codes over $\mathbb{F}_{5^4}$ provide considerable improvement in spectral efficiency of about 1.3 b/s/Hz when compared to MIMO-OFDM-IM with BPSK, in the case of 4×4 MIMO scenario. Moreover, the BER performance is observed to be similar. For a 2×2 system, the proposed FR-NSFBC-IM codes over $\mathbb{F}_{5^2}$ provide an improvement in spectral efficiency of about 0.9 b/s/Hz when compared to MIMO-OFDM-IM system, with Rate-1 Alamouti code and QPSK. The BER performance is observed to be similar in this case as well. Additionally, in case of a 2×2 MIMO system, the proposed codes provide an improvement in spectral efficiency by 0.6 b/s/Hz with SNR gain of 1 dB, when compared to QOSFBC based design.

Appendix

A.1. Proof of Proposition 1

For $k-$ free transform components, from the definition of IGFFT we have

$$u_i = (n \bmod q)^{-1}\left(U_{j_1}\beta^{-ij_1} + U_{j_2}\beta^{-ij_2} + \cdots + U_{j_k}\beta^{-ij_k}\right) ;\quad 0 \leq i \leq n-1 \tag{19}$$

Consider the $q-$ cyclotomic coset $[j_1] = \{j_1, j_1q, j_1q^2, \ldots, j_1q^{r_{j_1}-1}\}$. The associated conjugacy class is given by $\beta^{[j_1]} = \{\beta^{j_1}, \beta^{j_1q}, \beta^{j_1q^2}, \ldots, \beta^{j_1q^{(r_{j_1}-1)}}\}$. Since the conjugacy class is of size r_{j_1}, the associated minimal polynomial f_{j_1} will be of degree r_{j_1}. Let the reciprocal polynomial of f_{j_1} be give by $f_{j_1}^*$. The roots of this reciprocal polynomial will be the inverses of $\beta^{[j_1]}$, i.e. the elements from the set $\beta^{-[j_1]} = \{\beta^{-j_1}, \beta^{-j_1q}, \ldots, \beta^{-j_1q^{(r_{j_1}-1)}}\}$. According to the property of the reciprocal polynomials, the reciprocal polynomial of an irreducible polynomial is also irreducible [16]. Hence, the degree of $f_{j_1}^*$ is r_{j_1}. Similarly, the reciprocal polynomial $f_{j_k}^*$ of degree r_{j_k} represents irreducible polynomial of f_{j_k} having roots from the set $\beta^{-[j_k]} = \{\beta^{-j_k}, \beta^{-j_kq}, \ldots, \beta^{-j_kq^{(r_{j_k}-1)}}\}$.

According to Lemma 1, if $\left\{\beta^{-[j_1]} \cup \beta^{-[j_2]} \cup \cdots \cup \beta^{-[j_k]}\right\}$ is the union set of conjugacy classes then the minimal degree reciprocal polynomial f_j^* associated with this set, will be of degree e_j, specified as

$$f_j^* = b_{e_j}x^{e_j} + b_{e_j-1}x^{e_j-1} + b_{e_j-2}x^{e_j-2} + \cdots + b_1x + b_0 \tag{20}$$

Where, $b_i \in F_q$ for all $0 \leq i \leq e_j$ and not all $b_i = 0$ Since f_j^* is the reciprocal polynomial corresponding to $\left\{\beta^{-[j_1]} \cup \beta^{-[j_2]} \cup \cdots \cup \beta^{-[j_k]}\right\}$, we have $f_j^*(\beta^{-[j_1]}) = f_j^*(\beta^{-[j_2]}) = \cdots f_j^*(\beta^{-[j_k]}) = 0$. From (20), we then have.

$$\begin{aligned} \beta^{-e_jj_1} &= -\tfrac{1}{b_{e_j}}\textstyle\sum_{i=0}^{e_j-1} b_i\beta^{-ij_1}, \\ &\vdots \\ \beta^{-e_jj_k} &= -\tfrac{1}{b_{e_j}}\textstyle\sum_{i=0}^{e_j-1} b_i\beta^{-ij_k} \end{aligned} \tag{21}$$

Now consider (19) with $i = e_j + l$, then

$$a_{e_j+l} = (n \bmod q)^{-1}\left(U_{j_1}\beta^{-(e_j+l)j_1} + U_{j_2}\beta^{-(e_j+l)j_2} + \cdots + U_{j_k}\beta^{-(e_j+l)j_k}\right) \tag{22}$$

Since the term $\beta^{-(e_j+l)j_1} = \beta^{-(e_j)j_1}\beta^{-lj_1}$ and the condition $b_i' = -b_i/b_{e_j} \in \mathbb{F}_q$, we have

$$\beta^{-(e_j+l)j_1} = \beta^{-lj_1}\sum_{i=0}^{e_j-1} b_i'\beta^{-ij_1}$$

from (21).

Expanding the summation, we get

$$\begin{aligned} \beta^{-(e_j+l)j_1} &= \beta^{-lj_1}\{b_0' + b_1'\beta^{-j_1} + \cdots + b_{e_j-l}'\beta^{-(e_j-l)j_1} \\ &\quad + b_{e_j-l+1}'\beta^{-(e_j-l+1)j_1} + \cdots + b_{e_j-1}'\beta^{-(e_j-1)j_1}\} \\ \beta^{-(e_j+l)j_1} &= \{b_0'\beta^{-lj_1} + b_1'\beta^{-(l+1)j_1} + \cdots + b_{e_j-l}'\beta^{-e_jj_1} \\ &\quad + b_{e_j-l+1}'\beta^{-j_1}\beta^{-e_jj_1} + \cdots + b_{e_j-1}'\beta^{-(l-1)j_1}\beta^{-e_jj_1}\} \end{aligned}$$

Again, substituting for $\beta^{-e_jj_1}$ from (21), we get

$$\begin{aligned} \beta^{-(e_j+l)j_1} &= b_0'\beta^{-lj_1} + b_1'\beta^{-(l+1)j_1} + \cdots + b_{e_j-l}'\sum_{i=0}^{e_j-1} b_i'\beta^{-ij_1} + \cdots \\ &\quad + b_{e_j-1}'\beta^{-(l-1)j_1}\sum_{i=0}^{e_j-1} b_i'\beta^{-ij_1} \end{aligned} \tag{23}$$

From the above analysis, we can see that whenever the degree of β exceeds e_j, $\beta^{-(e_j-1)}$ can be substituted appropriately from (21). As the maximum degree term is $\beta^{-(e_j-1)}$, Eq. (23) can now be written as

$$\beta^{-(e_j+l)j_1} = \left\{v_0'' + v_1''\beta^{-j_1} + \cdots + v_{e_j-2}''\beta^{-(e_j-2)j_1} + v_{e_j-1}''\beta^{-(e_j-1)j_1}\right\} \tag{24}$$

Now, consider the term $\beta^{-(e_j+l)j_2}$. From (21), we see that the constants in the summation of $\beta^{-(e_j)j_1}$ $\beta^{-(e_j)j_2}$ are same. Hence, following (24), the term $\beta^{-(e_j+l)j_2}$ can be written as

$$\beta^{-(e_j+l)j_2} = \left\{v_0'' + v_1''\beta^{-j_2} + \cdots + v_{e_j-2}''\beta^{-(e_j-2)j_2} + v_{e_j-2}''\beta^{-(e_j-1)j_2}\right\} \tag{25}$$

In general,

$$\beta^{-(e_j+l)j_k} = \left\{v_0'' + v_1''\beta^{-j_k} + \cdots + v_{e_j-2}''\beta^{-(e_j-2)j_k} + v_{e_j-2}''\beta^{-(e_j-1)j_k}\right\} \tag{26}$$

Following (19) and (24)–(26) can be written as

$$u_{e_j+l} = (n \, mod \, q)^{-1} \times \left(U_{j_1} \sum_{i=0}^{e_j-1} v_i'' \beta^{-ij_1} + U_{j_2} \sum_{i=0}^{e_j-1} v_i'' \beta^{-ij_2} + \cdots + U_{j_k} \sum_{i=0}^{e_j-1} v_i'' \beta^{-ij_k} \right) \quad (27)$$

$$u_{e_j+l} = (n \, mod \, q)^{-1} \sum_{i=0}^{e_j-1} v_i'' \left(U_{j_1} \beta^{-ij_1} + U_{j_2} \beta^{-ij_2} + \cdots + U_{j_k} \beta^{-ij_k} \right) \quad (28)$$

The term inside the brackets is nothing but u_i. Hence, (28) can be written as

$$u_{e_j+l} = (n \, mod \, q)^{-1} \sum_{i=0}^{e_j-1} v_i'' u_i \quad (29)$$

From (29), we can infer that the element u_{e_j+l} for $0 \le l \le n - e_j - 1$ can be expressed as linear combination of first e_j elements of codeword vector $\mathbf{u}$. Where $v_i'' \in \mathbb{F}_q$. Therefore, $R_q(\mathbf{u})$ can now be given by the number of linearly independent elements in the set $\{u_0, u_1, \ldots, u_{e_j-1}\}$. It is to be noted that if $j_1, j_2, \ldots, j_k$ belong to different $q-$ cyclotomic cosets $[j_1]_n, [j_2]_n, \ldots, [j_k]_n$ then all roots correspond to different minimal polynomials $f_{j_1}, f_{j_2}, \ldots, f_{j_k}$. Hence, $e_j = r_{j_1} = r_{j_2} + \cdots + r_{j_k}$.

We now try to determine the rank of code $\mathscr{C}$ by finding the existence of codeword $\mathbf{u}$ with the number of linearly independent elements less than e_j.

A.2. Proof of Proposition 2

The IGFFT equation with $k-$ non-zero free transform components $\{U_{j_1}, U_{j_2}, \ldots, U_{j_k}\}$,(and rest of the components constrained to zero) is be given by,

$$u_i = (n \, mod \, q)^{-1} (U_{j_1} \alpha^{-ij_1} + U_{j_2} \alpha^{-ij_2} + \cdots + U_{j_k} \alpha^{-ij_k}), \; 0 \le i \le n-1 \quad (30)$$

Following Proposition 1, each codeword vector has elements related by,

$$\sum_{i=0}^{e_j} v_i u_i = 0 \; ; \qquad v_i \in \mathbb{F}_q \quad (31)$$

Since it is evident that the components of a codeword vector – starting from location e_j – are a linear combination of first e_j components, the rank of the codeword vector $\mathbf{u}$ (equivalently codeword matrix $\mathbf{U}$) is determined by the first e_j components of the codeword vector. We now try to find the minimum rank of the code $\mathscr{C}$ by finding the codeword vector with a minimum number of linearly independent components within the first e_j elements $\{u_o, u_1, \ldots, u_{e_j-1}\}$ of the codeword vector $\mathbf{u}$. We first consider the cyclic code $\mathscr{C}$ obtained using three transform components and generalize it the case of k free transform components. Without loss of generality let us consider the three free transform components to be $U_{j_1}, U_{j_2}, U_{j_3}$. (2) now becomes

$$u_i = (n \, mod \, q)^{-1} (U_{j_1} \beta^{-ij_1} + U_{j_2} \beta^{-ij_2} + U_{j_3} \beta^{-ij_3}), \; 0 \le i \le n-1 \quad (32)$$

From (32) in (31), we get

$$(n \, mod \, q)^{-1} \sum_{i=0}^{e_j} v_i (U_{j_1} \alpha^{-ij_1} + U_{j_2} \alpha^{-ij_2} + U_{j_3} \alpha^{-ij_3}) = 0 \quad (33)$$

Since the term $(n \; mod \; q)^{-1}$ is a constant, the above (33) be rewritten as

$$U_{j_1} \sum_{i=0}^{e_j} v_i \beta^{-ij_1} + U_{j_2} \sum_{i=0}^{e_j} v_i \beta^{-ij_2} + U_{j_3} \sum_{i=0}^{e_j} v_i \beta^{-ij_3} = 0 \quad (34)$$

Case 1: Let us consider the transform component indices as $j_1, j_2 = j_1 q^{s_1}, j_3 = j_1 q^{s_2}$, chosen from cyclotomic coset $[j_1]$ of size r_{j_1}. Then according to Proposition 1 $e_j = r_{j_1}$. (34) now becomes

$$U_{j_1} \sum_{i=0}^{r_{j_1}} v_i \beta^{-ij_1} + U_{j_1 q^{s_1}} \sum_{i=0}^{r_{j_1}} v_i \beta^{-ij_1 q^{s_1}} + U_{j_1 q^{s_2}} \sum_{i=0}^{r_{j_1}} v_i \beta^{-ij_1 q^{s_2}} = 0 \quad (35)$$

From (35), we have

$$U_{j_1} \sum_{i=0}^{r_{j_1}} v_i \beta^{-ij_1} + U_{j_1 q^{s_1}} \left(\sum_{i=0}^{r_{j_1}} v_i \beta^{-ij_1} \right)^{q^{s_1}} + U_{j_1 q^{s_2}} \left(\sum_{i=0}^{r_{j_1}} v_i \beta^{-ij_1} \right)^{q^{s_2}} = 0 \quad (36)$$

The above (36) can be rewritten as

$$\sum_{i=0}^{r_{j_1}} v_i \beta^{-ij_1} + \frac{U_{j_1 q^{s_1}}}{U_{j_1}} \left(\sum_{i=0}^{r_{j_1}} v_i \beta^{-ij_1} \right)^{q^{s_1}} + \frac{U_{j_1 q^{s_2}}}{U_{j_1}} \left(\sum_{i=0}^{r_{j_1}} v_i \beta^{-ij_2} \right)^{q^{s_2}} = 0 \quad (37)$$

Let us consider a codeword vector $\mathbf{u}$ obtained by the free transform component U_{j_1} such that $U_{j_1}{}^{-1} = x \in \mathbb{F}_{q^g}$, where $g | r_{j_1} | m$. The number of linearly independent elements of this $\mathbf{u}$ is now represented by $r_u \le r_{j_1}$, With this, (37) can now be written as

$$\sum_{i=0}^{r_u} v_i \beta^{-ij_1} + x U_{j_1 q^{s_1}} \left(\sum_{i=0}^{r_u} v_i \beta^{-ij_1} \right)^{q^{s_1}} + x U_{j_1 q^{s_2}} \left(\sum_{i=0}^{r_u} v_i \beta^{-ij_2} \right)^{q^{s_2}} = 0 \quad (38)$$

Since, we are determining the value of r_u, we now consider the case for which $r_u < r_{j_1}$ can be obtained. Let us define $\delta_i = U_{j_1 q^{s_i}} \left(\sum_{i=0}^{r_a} v_i \beta^{-ij_1} \right)^{q^{s_i}}$ for $i = 1, 2$. Also, since $x \in \mathbb{F}_{q^g}$ the element x can be expressed as $\sum_{i=0}^{g-1} d_i x_i$, with $d_i \in \mathbb{F}_q$. Here, $\{x_0, x_1, \ldots, x_{g-1}\}$ forms trivial basis of $\mathbb{F}_{q^g}$. (38) can now be written as

$$\sum_{i=0}^{r_a} v_i \beta^{-ij_1} + \delta_1 \sum_{i=0}^{g-1} d_i x_i + \delta_2 \sum_{i=0}^{g-1} d_i x_i = 0 \quad (39)$$

We can see that δ_i is dependent on the value of $U_{j_1 q^{s_i}}$ for $i = 1, 2$. Hence, there can exist $\delta_1, \delta_2 \notin \mathbb{F}_{q^g}$, and $\delta_1 \ne \delta_2$ such that the set $\{\delta_1 x_0, \ldots, \delta_1 x_{g-1}, \delta_2 x_0, \ldots, \delta_2 x_{g-1}, \}$ forms the set of $2g$ linearly independent elements in the field $\mathbb{F}_{q^m}$. Since, $\{\beta^{-j_1}, \beta^{-2j_1}, \ldots \beta^{-r_{j_1} j_1}\}$ forms the set of linearly independent elements in $\mathbb{F}_{q^m}$, the set $\{\beta^{-j_1}, \beta^{-2j_1}, \ldots \beta^{-(r_{j_1}-2g)j_1}, \delta_1 x_0, \ldots, \delta_1 x_{g-1}, \delta_2 x_0, \ldots, \delta_2 x_{g-1}\}$ forms the set of linearly independent elements in $\mathbb{F}_{q^m}$. This implies that the value of r_u in the first summation of (38) is $r_u = r_{j_1} - 2g$. The vector $\mathbf{u}$ obtained using the above considerations is indeed the vector with minimum number of linearly independent elements because $\mathbf{u}$ obtained for any other values of $U_{jq^{s_1}}, U_{jq^{s_2}}, \delta_1 x_i = \delta_2 x_j$ for some i, j, the set $\{\delta_1 x_0, \delta_1 x_1, \ldots, \delta_1 x_{g-1}\} \cap \{\delta_2 x_0, \delta_2 x_1, \ldots, \delta_2 x_{g-1}\}$ is not a null set. Hence, the value of r_u required to satisfy (38) is $r_u > r_{j_1} - 2g$. Hence, $R_q(\mathscr{C}) = r_{j_1} - 2g$.

In general, if $\mathscr{C}$ is designed using k free transform coefficients with indices chosen from same $q-$ cyclotomic coset of size $r_{j_1} > k$ then $R_q(\mathscr{C}) = r_{j_1} - (k-1)g$. **Note**: In case of $r_{j_1} = m$, $R_q(\mathscr{C}) = m - (k-1)g$. **Case 2**: Without loss of generality, let us consider the free transform components to be $U_{j_1}, U_{j_2}, U_{j_3}$ with transform component indices j_1, j_2, j_3 chosen from different $q-$ cyclotomic cosets $[j_1], [j_2], [j_3]$ of sizes r_{j_1}, r_{j_2} and, r_{j_3} respectively.

According to Proposition 1, the minimum degree irreducible polynomial f_j which contains $\beta^{-j_1}, \beta^{-j_2}, \beta^{-j_3}$ as roots is of degree

$e_j = r_{j_1} + r_{j_2} + r_{j_3}$. (34) now becomes

$$U_{j_1} \sum_{i=0}^{r_{j_1}+r_{j_2}+r_{j_3}} v_i \beta^{-ij_1} + U_{j_2} \sum_{i=0}^{r_{j_1}+r_{j_2}+r_{j_3}} v_i \beta^{-ij_2} + U_{j_3} \sum_{i=0}^{r_{j_1}+r_{j_2}+r_{j_3}} v_i \beta^{-ij_k} = 0 \tag{40}$$

Which can be simplified to

$$\sum_{i=0}^{r_{j_1}+r_{j_2}+r_{j_3}} v_i \beta^{-ij_1} + \frac{U_{j_2}}{U_{j_1}} \sum_{i=0}^{r_{j_1}+r_{j_2}+r_{j_3}} v_i \beta^{-ij_2} + \frac{U_{j_3}}{U_{j_1}} \sum_{i=0}^{r_{j_1}+r_{j_2}+r_{j_3}} v_i \beta^{-ij_k} = 0 \tag{41}$$

Since, $U_{j_1}, U_{j_2}, U_{j_3} \in \mathbb{F}_{q^m}$ take on values independently let us consider a codeword vector $\mathbf{u}$, obtained by considering $U_{j_1}{}^{-1} = x \in \mathbb{F}_{q^g}$. Also, let $r_u \leq r_{j_1}$ be the number of linearly independent elements of this codeword vector $\mathbf{u}$. From Proposition 1, these r_u elements are the first set of coefficients $\{u_0, u_1, \ldots, u_{r_u-1}\}$ of the codeword vector $\mathbf{u}$. The remaining $n - r_u$ elements of this codeword are linearly dependent on the first r_u elements. With this consideration (41) can be written as

$$\sum_{i=0}^{r_u} a_i \alpha^{-ij_1} + xU_{j_2} \left(\sum_{i=0}^{r_u} v_i \beta^{-ij_1} \right) + xU_{j_3} \left(\sum_{i=0}^{r_u} v_i \beta^{-ij_2} \right) = 0 \tag{42}$$

Let us define $\delta_i = U_{j_i} \left(\sum_{i=0}^{r_u} v_i \beta^{-ij_1} \right)$ for $i = 1, 2$. Also, since $x \in \mathbb{F}_{q^g}$ the element x can be expressed as $\sum_{i=0}^{g-1} d_i x_i$, with $d_i \in \mathbb{F}_q$. Here, $\{x_0, x_1, \ldots, x_{g-1}\}$ forms trivial basis of $\mathbb{F}_{q^g}$. Using this (42) can now be written as

$$\sum_{i=0}^{r_u} v_i \beta^{-ij_1} + \delta_1 \sum_{i=0}^{g-1} d_i x_i + \delta_2 \sum_{i=0}^{g-1} d_i x_i = 0 \tag{43}$$

We can see that δ_i is dependent on the value of U_{j_i} for $i = 1, 2$. Hence there can exist $\delta_1, \delta_2 \notin \mathbb{F}_{q^g}$, and $\delta_1 \neq \delta_2$ such that the set $\{\delta_1 x_0, \ldots, \delta_1 x_{g-1}, \delta_2 x_0, \ldots, \delta_2 x_{g-1}, \}$ forms the set of $2g$ linearly independent elements in the field $\mathbb{F}_{q^m}$.

Eq. (31) is obtained from the result that $e_j + 1$ elements of any codeword $\mathscr{C}$ are linearly dependent. This means that there exists a minimum of $e_j + 1$ terms in the summation of (31) for the summation to be 0. In (43), we have identified 2g elements which are linearly independent. From a close examination of (43), we can write $r_u + 1 + 2g = r_{j_1} + r_{j_2} + r_{j_3} + 1 \implies r_u = r_{j_1} + r_{j_2} + r_{j_3} - 2g$. It is to be noted that, for the case $\delta_1 x_i = \delta_2 x_j$ for some i, j the set $\{\delta_1 x_0, \delta_1, \ldots, \delta_1 x_{g-1}\} \cap \{\delta_2 x_0, \delta_2 x_1, \ldots, \delta_2 x_{g-1}\}$ is not a null set. Hence, the value of r_a required to satisfy (31) is $r_u > r_{j_1} + r_{j_2} + r_{j_3} - 2g$. Hence, the upper limit in the summation for which the (31) holds good is $r_{j_1} + r_{j_2} + r_{j_3} - 2g$. That is the first $e_j - 2g = r_{j_1} + r_{j_2} + r_{j_3} - 2g$ elements are linearly independent and the rest will be dependent on these e_j elements. When viewed as $m \times n$ matrices over $\mathbb{F}_q$ this means the first $e_j - 2g = r_{j_1} + r_{j_2} + r_{j_3} - 2g$ columns are linearly independent and rest of the columns are linearly dependent on these columns. Here we can see that depending on the values of $r_{j_1}, r_{j_2}, r_{j_3}$ the value of e_j can be greater than m or less than m. Hence, following rank nullity theorem rank of these codewords is equal to $min(e_j, m)$. Since code $\mathscr{C}$ contains codewords for all possible values of $U_{j_1}, U_{j_2}, U_{j_3}$, $R_q(\mathscr{C}) = min(r_{j_1}, r_{j_2}, r_{j_3}, m)$. In general for $\mathscr{C}$ constructed using k free transform coefficients with indices coming from k different $q-$ cyclotomic cosets then $R_q(\mathscr{C}) = min(r_{min}, m)$ where $r_{min} = min(r_{j_1}, r_{j_2}, \ldots, r_{j_k})$.

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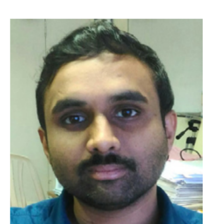

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