Analysis of Mobile Beacon aided In-Range Localization Scheme in Ad Hoc Wireless Sensor Networks

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ABSTRACT

In this paper, We mathematically model the In-Range localization scheme in the presence of a Mobile Beacon. In the In-Range localization scheme, a sensor with unknown location is localized to a disc centered at the position of the beacon, if the sensor under consideration can successfully decode a transmission from the beacon. In our approach a Mobile Beacon guided by a mobility model is used to generate the virtual beacons, there by eliminating the need to deploy static beacons that are required in the classical In-Range localization scheme. For analysis, we consider a Mobile Beacon guided by the Random Way Point (RWP) mobility model with In-Range localization scheme. The main contribution of this paper consists of mathematical models for the In-Range localization parameters in the presence of a Mobile Beacon guided by the RWP mobility model.

Categories and Subject Descriptors

A.1 [**Algorithms**]: Sensor Networks

General Terms

Algorithms

Keywords

Wireless Sensor Networks, Localization, In-Range Localization, Mobile Beacon, Mobility Models, Random Way Point

1. INTRODUCTION

Spatial or location information is of intrinsic interest in sensor networks; for example, sensors use it in data combining and estimation. However, such information can neither be pre-configured in sensors owing to their ad hoc and possibly random deployment nor can it be centrally disseminated to sensors because of absence of a centralized coordinator. Thus, it is imperative that sensors infer their locations autonomously using low cost infrastructure.

Localization is a process that enables nodes of the sensor network to compute their locations. One possible solution for solving the localization problem is manual configuration; but this is highly impossible in large scale deployments. Another possible solution is to equip the sensor nodes with GPS receivers, however it is not considered to be an economically feasible solution. There are other methods known as beacon based approaches for localization in which a few sensors known as beacons[1] would aid the process of localization of sensors with unknown locations. [2] discusses one such beacon based approach that uses In-Range localization with static beacons. [2] also discusses the drawbacks of localization using traditional ranging techniques over In-Range localization. Although the beacon based approaches reduce the cost of sensor network to a great extent, the cost of beacons is still a major component in the total cost. Moreover, the beacons will have no role to play once they have transmitted their location information and once their neighbors have decoded the beacon information for localization.

This motivates us to design a localization mechanism which uses In-Range localization in the presence of a Mobile Beacon. The Mobile Beacon would generate virtual beacons required for In-Range localization. We use RWP mobility model to guide the motion of the Mobile Beacon because it is simple to implement in the real world.

The rest of the paper is organized as follows. Section 2 discusses the Related Work. In Section 3, we discuss the RWP mobility model. In section 4, we give an overview of the In-Range Localization Scheme in the presence of a Mobile Beacon. Section 5 deals with Mobile Beacon aided One Dimensional Localization Process. We provide the Localization Mechanism in Section 6. In Section 7, Numerical Results are discussed. Conclusions are given in Section 8. Section 9 introduces the Future Research Work. Finally, the Proofs are presented in Section 10.

*Strict reverse alphabetical ordering of names is followed; Hence both are first authors.

[1]A special node in the sensor network that knows its own location.

2. RELATED WORK

[2] introduces the In-Range localization scheme that requires the deployment of beacons along with sensors. It also discusses the advantages of the In-Range localization over other existing localization techniques. The effect of static beacons in the In-Range localization scheme can be equivalently produced by using a single Mobile Beacon. Thus the use of a single Mobile Beacon eliminates the need for deploying static beacons.

Recently, few schemes [3] [4] [5] [6] have been proposed that employs Mobile Beacons for localizing sensor networks. [3] uses four GPS equipped Mobile Beacons, which coordinate based on distance estimates using RSSI for localizing the sensor network. [4] uses a single Mobile Beacon and depends on RSSI for estimating distance between sensor nodes and the current position of the Mobile Beacon. Since both of these schemes are RSSI based, they have disadvantages as discussed in [1]. [5] proposes a localization scheme using a Mobile Beacon based on TOA, TOA also has disadvantages as discussed in [1].

[6] describes MAL, a mobile-assisted localization method which employs a mobile user to assist in measuring distances between node pairs until these distance constraints form a globally rigid structure that guarantees a unique localization. This approach involves measuring distances to nodes from various positions of the mobile user. Hence it also has disadvantages as discussed in [1].

3. RWP MOBILITY MODEL

RWP mobility model guides the motion of the Mobile Beacon in the sensor field. The Mobile Beacon moves through a series of points (p_1, p_2, ..., p_k) selected from the deployment area with uniform probability. The Mobile Beacon moves from p_i to p_{i+1} with a speed, V_i selected from $[V_{min}, V_{max}]$ with uniform probability. The pause time of the Mobile Beacon at p_{i+1} is given by

$$t_c - \frac{d(p_i, p_{i+1})}{V_i}$$

where,
$d(p_i, p_j)$ represents euclidian distance between the points p_i and p_j.
t_c is a constant $\geq \frac{max(d(p_i,p_j))}{V_{min}}$ for all p_i and p_j in the deployment area.

With this definition for pause times of RWP, the virtual beacons arrive with a uniform rate, $\frac{1}{t_c}$.

We define a term called **Mobile Beacon Cardinality,** $M(t)$. It represents the cardinality of the set of points visited by the Mobile Beacon till time t. Mathematically,

$$M(t) = 1 + \lfloor \frac{t}{t_c} \rfloor$$

4. IN-RANGE LOCALIZATION IN PRESENCE OF A MOBILE BEACON

Consider a randomly deployed sensor network in a geographical region $\mathcal{A}$; in this paper $\mathcal{A} \subset \mathbb{R}$. The sensors are indexed by $i\epsilon\{1, 2, ..., N\}$(N being the number of sensors in the deployment area) and the virtual beacons by $i\epsilon\{N+1, N+2, ..., N+M(t)\}$ at any instant of time t. We say that a transmission can be "decoded" by a sensor when its signal to interference ratio (SIR) exceeds a given threshold β. The *transmission-range* is then defined as the maximum distance at which a receiver can decode a transmitter in the absence of any co-channel interference. We denote the transmission range of sensors and that of the Mobile Beacon by R_0. The sensors within a distance of R_0 from i will be called its neighbors. The set of neighbors of i will be denoted by N_i and their count by n_i. By the location of sensor we mean its co-ordinates and denote it compactly by v_i; in this paper v_i is just the x-coordinate of sensor i.

A localization set for a sensor i is a subset of the region of deployment. Let $X_i(t, n)$ denote the localization set for the sensor i at time t after n iterations of In-Range localization. Thus the initial localization set, $X_i(0, 0) = \mathcal{A}$ for all $i\epsilon\{1, 2, .., N\}$. $D(v, r)$ denotes a disk of radius r centered at v; in one dimension disks are replaced by intervals. O denotes the origin. If G and H are two sets, $G + H$ denotes the set addition, i.e., $G + H = \{g + h|g\epsilon G, h\epsilon H\}$.

We define a term called **Node Density**, $\lambda(t)$. It represents the total number of virtual beacons and sensors present in unit length of the deployment area at any instant of time, t. Mathematically,

$$\lambda(t) = \frac{N + M(t)}{A}$$

in one-dimensional sensor networks, A represents the length of the deployment region.

We define another term called **Sensor Fraction**, K. It represents the fraction of sensors present in the pool of sensors and the Mobile Beacon in the deployment area; Mathematically,

$$K = \frac{N}{N+1}$$

The number of iterations of In-Range localization carried out as soon as a virtual beacon is produced by a Mobile Beacon is referred to as **Iteration Constant**.

The following gives the iterative scheme for In-Range localization. n is the number of iterations and t is the time. For $n \geq 0$, $t \geq 0$ and $i = 1, 2, ..., N$.

$$Y_i(t, n+1) = \bigcap_{k\epsilon N_i} (X_k(t, n) + D(0, R_0)) \quad (1)$$

$$X_i(t, n+1) = X_i(t, n) \cap Y_i(t, n+1) \quad (2)$$

If i is in the range of k, i is certainly in the region $(X_k(t, n)+D(0, R_0))$. Since this property holds for each neighbor of i, i is localized to $\bigcap_{k\epsilon N_i}(X_k(t, n)+D(0, R_0))$. Thus, it follows that at any instant of time t, the localization set of i after $(n + 1)$ iterations is the intersection of its localization set after n iterations and $\bigcap_{k\epsilon N_i}(X_k(t, n) + D(0, R_0))$.

Let $\mathscr{L}(X)$ denote a measure of set X; in one dimension it is the length of X. Define, $\chi_i(t, n) = \mathscr{L}(X_i(t, n))$, which we call the *localization error* of sensor i in time t

and iteration n. $\chi_i(t,n) = 0$ for all virtual beacons. Let $\underline{\chi}(t,n) = (\chi_1(t,n), \chi_2(t,n), ..., \chi_N(t,n))$ and consider the vector valued process $\{\underline{\chi}(t,n); n \geq 0, t \geq 0\}$ which we call the *localization process*. Note from (2) that for each i, $\chi_i(t,n)$ is non-increasing with t and n.

The performance measures which are of interest include

- $\overline{\chi}(t,n) = \frac{1}{N+1}\sum_{i=1}^{N}\chi_i(t,n)$
- $v(t,n) = \frac{1+\sum_{i=1}^{N} 1_{\{\chi_i(t,n)<A\}}}{N+1}$, where $1_{\{.\}}$ denotes the indicator function.

Thus by definition, $\overline{\chi}(t,n)$ is the average localization error in the network at the time t with iteration constant n. $v(t,n)$ is the fraction of nodes localized at the instant of time t with iteration constant n.

5. MOBILE BEACON AIDED ONE DIMENSIONAL LOCALIZATION PROCESS

We assume N to be very large and model the random dispersion of sensors on the real line as a one dimensional poisson point process Ψ of intensity λ_s; poisson points indicate the locations of sensing nodes. The field of deployment is assumed to be $[-\frac{A}{2}, \frac{A}{2}]$; in other words A represents the length of deployment area. The Mobile Beacon starts from an initial location x_1 and then selects a point (say x_2) in $[-\frac{A}{2}, \frac{A}{2}]$ with a uniform probability. Then the Mobile Beacon moves to the location x_2 with a velocity V_1, chosen from $[V_{min}, V_{max}]$ with uniform probability. At the point x_2, the Mobile Beacon pauses for a time given by $t_c - \frac{x_2 - x_1}{V_1}$. The Mobile Beacon would broadcast its location information and then it is ready to select a new destination x_3. The process continues in this way aiding the generation of virtual beacons.

We assume that each sensor would maintain a list of neighbors. When the Mobile Beacon broadcasts location information at x_i, the sensors within the transmission range of the Mobile Beacon would add the current location of Mobile Beacon i.e., x_i to their neighbor list, thus generating virtual beacons.

Disk $D(v_j, R_0)$ in $\mathbb{R}$ is the interval of length $2R_0$ centered at v_j; v_j is simply x co-ordinate of j. Thus $D(v_j, R_0)$ extends from $v_j - R_0$ to $v_j + R_0$. $\chi_j(t,n)$ denotes the length of $X_j(t,n)$. The length of the interval lying to the right of j is denoted by $\Delta_j^r(t,n)$ while the length of the interval lying to the left of j is denoted by $\Delta_j^l(t,n)$; hence $\chi_j(t,n) = \Delta_j^l(t,n) + \Delta_j^r(t,n)$.

For each sensor j, $\Delta_j^l(0,0) = \Delta_j^r(0,0) = \frac{A}{2}$ and for virtual beacons, these values are always 0. In the point process model, A should be interpreted as the initial uncertainty preset in each sensor. The evolution (2) in this setting is as follows. Recall that N_j denotes the set of neighbors of j. For $n \geq 1$ and $j = 1, 2, ..., N$.

$$u_j^r(t,n) = \arg\min(v_k + \Delta_k^r(t,n-1)) \tag{3}$$

$$\begin{aligned}\Delta_k^r(t,n) = \min(&\Delta_k^r(t,n-1), \\ &v_{u_j^r(t,n)} + \Delta_{u_j^r(t,n)}^r(t,n-1) + R_0 - v_j)\end{aligned} \tag{4}$$

$$u_j^l(t,n) = \arg\max(v_k - \Delta_k^l(t,n-1)) \tag{5}$$

$$\begin{aligned}\Delta_k^l(t,n) = \max(&\Delta_k^l(t,n-1), \\ &v_j - v_{u_j^l(t,n)} + \Delta_{u_j^l(t,n)}^l(t,n-1) + R_0)\end{aligned} \tag{6}$$

$$\chi_j(t,n) = \Delta_j^r(t,n) + \Delta_j^l(t,n) \tag{7}$$

If N_j is empty, then by convention, the minimum over an empty set is taken to be ∞ and we define the location of $u_j^r(t,n+1)$ to be ∞. Similarly for $u_j^l(t,n+1)$.

To understand the iterative process given by (3), let us first consider $n = 1$. Assume that j is a sensor. $\Delta_k^r(0,0) = \frac{A}{2}$ and $X_j(t,1)$ is decided only by the beacons in its range. Further, $X_j(t,1)$ will be determined by the leftmost and the rightmost beacon in the range of j; the leftmost beacon will determine $\Delta_k^r(t,1)$ and the rightmost beacon will determine $\Delta_k^l(t,1)$. Step (3) locates the leftmost beacon; it is denoted by $u_j^r(t,1)$. Then it is easy to see that $\Delta_k^r(t,1) = v_{u_j^r(t,1)} + R_0 - v_j$ since R_0 is the range and v_j is j's location. Similarly $u_j^l(t,1)$ denotes the rightmost beacon so that $\Delta_k^l(t,1) = v_j - v_{u_j^l(t,1)} + R_0$.

Now for $n \geq 2$, consider a situation in which j has only one neighbor denoted by s_1 with location v_1. Suppose that s_1 is a sensor. Further, $n-1$ iterations are over; j has been localized to $[v_j - \Delta_j^l(t,n-1), v_j + \Delta_j^r(t,n-1)]$. Since j lies in the transmission range of s_1 by virtue of this single constraint, j must lies within $[v_1 - \Delta_1^l(t,n-1) - R_0, v_1 + \Delta_1^r(t,n-1) + R_0]$. The length of the "right" side of this interval is $v_1 + \Delta_1^r(t,n-1) + R_0 - v_j$. It follows that $\Delta_j^r(t,n)$ will be the minimum of $\Delta_j^r(t,n-1)$ and $v_1 + \Delta_1^r(t,n-1) + R_0 - v_j$. Similar analysis applies to $\Delta_j^l(t,n)$. Equations (3) and (4) simply extend this logic to a general case.

Though (3) and (4) are much simplified compared to (2), $\underline{\chi}(t,n)$ is still not amenable to analysis. We now work with a typical point of the poisson process, called the tagged node (denoted by o) and study the "marginal" process of $\underline{\chi}(t,n)$ i.e., $\{\chi_o(t,n) | n \geq 0, t \geq 0\}$, the sequence of localization errors of the tagged node. The performance measures discussed in Section IV can be obtained at the tagged node as, $\overline{\chi}(t,n) = E\chi_o(t,n)$ and $v(t,n) = P(\chi_o(t,n) < A)$

5.1 The Marginal Process, $\{\chi_o(t,n) | n \geq 0, t \geq 0\}$

Since o is a sensor, $X_o(t,0) = \mathcal{A}$. The value of $X_0(t,1)$ is decided only by the beacons in the range of o. It is thus possible to explicitly characterize the distribution of $\chi_o(t,1)$. However, for further analysis we will work with a simpler process, $\{\Delta_o^r(t,n); n \geq 0, t \geq 0\}$. Note that $\Delta_o^r(t,1)$ and $\Delta_o^l(t,1)$ are identically distributed though not independent. By symmetry, this property holds for $n \geq 2$.

PROPOSITION 5.1. *The probability distribution function of $\Delta_o^r(t,1)$ is,*

$$P(\Delta_o^r(t,1) \leq y) = \begin{cases} 1 & \text{if } y > \frac{A}{2} \\ 1 - (1 - \frac{2R_0}{A})^{M(t)} & \text{if } 2R_0 < y \leq \frac{A}{2} \\ 1 - (1 - \frac{y}{A})^{M(t)} & \text{if } 0 < y \leq 2R_0 \end{cases}$$

The probability mass at $\frac{A}{2}$ is

$$P(\Delta_o^r(t,1) = \frac{A}{2}) = (1 - \frac{2R_0}{A})^{M(t)}$$

.

PROOF. See Section 10

□

COROLLARY 5.1.

$$E\Delta_o^r(t) = K[\frac{2R_0^2M(t)}{A}(1-\frac{2R_0}{A})^{M(t)-1}+ (1-\frac{2R_0}{A})^{M(t)} - 1 + \frac{A}{2}(1-\frac{2R_0}{A})^{M(t)}]$$

Observe from (3) that, for a given $\Delta_o^r(t,1)$, $\{\Delta_o^r(t,n), n \geq 2\}$ is determined by the (ordinary) sensors in the range of o. Let N_o^s denote the set of sensors in the range of o and n_o^s their number. Consider now the iteration (3) applied to o for $n \geq 2$.

$$u_o^r(t,n) = \arg\min_{k\epsilon N_o^s}(v_k + \Delta_k^r(t,n-1)) \quad (8)$$

and

$$\Delta_o^r(t,n) = \min(\Delta_o^r(t,n-1), v_{u_o^r(t,n)} + \Delta_{u_o^r(t,n)}^r(t,n-1)+R_0) \quad (9)$$

Recall that if N_o^s is empty, by convention minimum over N_o^s in (8) is infinite and $\Delta_o^r(t,n) = \Delta_o^r(t,n-1)$. Now a direct analysis of (9) amounts to analyzing $\{\underline{\chi}_o(t,n)|n \geq 0, t \geq 0\}$ since to find the probability distribution of $\Delta_o^r(t,n)$, we need the joint distribution of $\Delta_k^r(t,n-1)$, $k\epsilon N_o^s$. However an asymptotically exact approximation for the sequence $\{\chi_o(t,n)|t \geq 0, n \geq 0\}$ can be obtained as follows.

Since o is a typical point of poisson process, n_o^s is poisson distributed with mean $\lambda_s 2R_0$. We denote by o_k the k^{th} "sensor-neighbor" of o. For a given n_o^s, v_{o_k} are independent uniformly distributed random variables in $[-R_0, R_0]$. We now index these neighbors by $i\epsilon\{1,...,n_o^s\}$ based on the order statistics of v_{o_k} i.e., the sensor corresponding i^{th} smallest value of v_{o_k}'s is indexed i. Thus 1 (arg $min_{1\leq k\leq n_o^s} v_{o_k}$) is the leftmost neighbor and the rest in the increasing order towards right. Location of is denoted by v_i;

$$f_{v_1}(x|n_o^s = m) = \frac{m}{2R_0}(1-\frac{x+R}{2R_0})^{m-1}[1]$$

Now consider a sequence $\{\hat{\Delta}_o^r(t,n)|t \geq 0, n \geq 0\}$ such that $\hat{\Delta}_o^r(t,1) = \Delta_o^r(t,1)$ and for $n \geq 2$,

$$\hat{\Delta}_o^r(t,n) = min(\hat{\Delta}_o^r(t,1), v_1 + \hat{\Delta}_o^r(t,n-1) + R_0) \quad (10)$$

Thus $\{\hat{\Delta}_o^r(t,n), n \geq 2, t \geq 0\}$ can be generated iteratively; computation of the statistics of $\hat{\Delta}_o^r(t,n)$ requires only the statistics of $\hat{\Delta}_o^r(t,1)$ and $\hat{\Delta}_o^r(t,n-1)$ computed in the previous iteration. Let $F_{\Delta_o^r(t,n)}(x)$ and $F_{\hat{\Delta}_o^r(t,n)}(x)$ denote the cumulative probability distribution of $\Delta_o^r(t,n)$ and $\hat{\Delta}_o^r(t,n)$ respectively. Then the following holds.

PROPOSITION 5.2.

$$lim_{\lambda(t)\to\infty}|F_{\Delta_o^r(t,n)}(x) - F_{\hat{\Delta}_o^r(t,n)}(x)| = 0 \quad (11)$$

Since the underlying process for sensor distribution is poisson, the equations (9), (10) and (11) would follow from the discussion presented in [1].

5.2 The Marginal Process, $\{\ v(t,n),\ t \geq 0$ and $n \geq 0\}$

Recall that $v(t,n) = P(\chi_o(t,n) < A)$, the fraction of the nodes that get localized by the time t with iteration constant n.

PROPOSITION 5.3.

$$v(t,1) = (1-K) + K(1-(1-\frac{2R_0}{A})^{M(t)}) \quad (12)$$

and

$$lim_{\lambda(t)\to\infty}|(1-K) + K(1-(1-\frac{2nR_0}{A})^{M(t)}) - v(t,n)| = 0 \quad (13)$$

PROOF. See Section 10.

□

6. IN-RANGE LOCALIZATION ALGORITHM USING A MOBILE BEACON

Algorithm 1: In-Range localization in the presence of a Mobile Beacon

1. Select a mobility that guides the motion of the Mobile Beacon.
2. Select an initial point (source) for the Mobile Beacon.
3. Select the next point (destination) as defined by the mobility model.
4. The Mobile Beacon would move from the source to the destination with a velocity as defined by the chosen mobility model.
5. The Mobile Beacon would broadcast the current location information to the sensors and the sensors in the transmission range of the Mobile Beacon would add the current location of the Mobile Beacon to their neighbor list (flags this entry as a virtual beacon).
6. Execute the In-Range localization algorithm at each sensor and also compute the values of localization error and percentage of nodes localized.
7. Repeat steps 3-7 till the desired accuracy in terms of localization parameters is obtained.

7. RESULTS AND DISCUSSION

In the simulation, we take λ_s equal to 1 per m. A is obtained as follows. Since we generate 1000 poisson points for the random sensor placement model, the initial uncertainly for each sensor is then the expected length of this placement, i.e., $A = \frac{1000}{\lambda_s}$ and the initial location is its center.

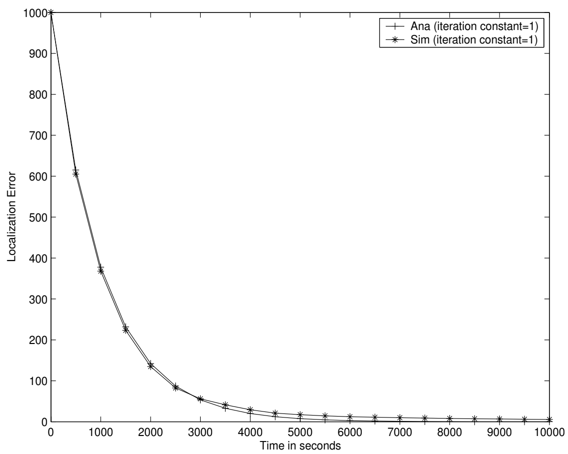

Figure 1: Variation of localization error with time - Analysis and Simulation

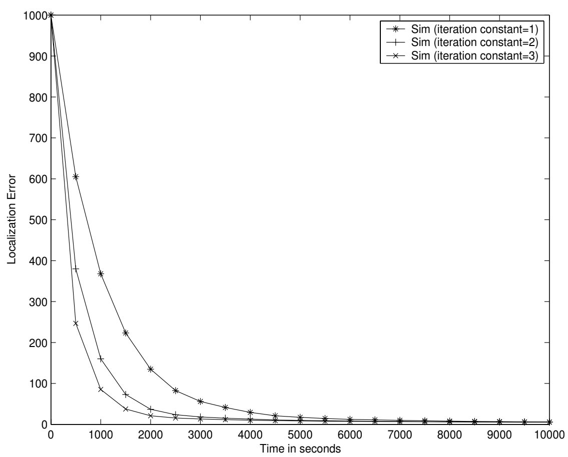

Figure 2: Variation of localization error with time - Simulation with iteration constants 1, 2 and 3

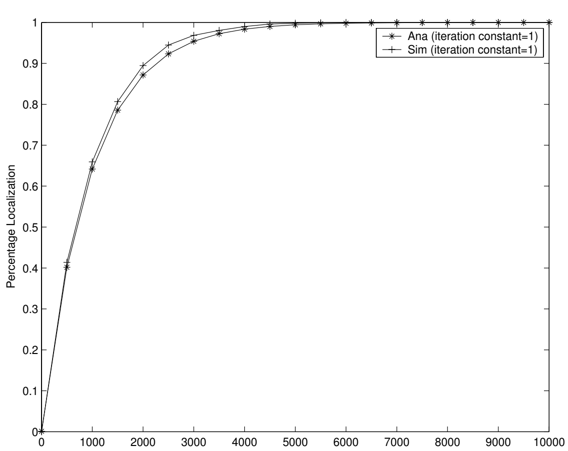

Figure 3: Variation of percentage of localization with time - Analysis and Simulation

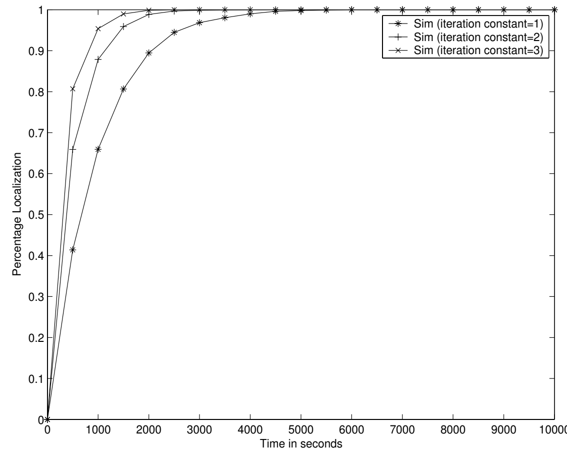

Figure 4: Variation of percentage of localization with time - Simulation with iteration constants 1, 2 and 3

The transmission range of the Mobile Beacon and that of the sensors is taken as 25 units with the value of constant t_c equal to 50 seconds.

Figure 1 shows the variation of average localization error with the time (in seconds). For the iteration constant equal to 1, the analytical results match extremely well with the simulation results. For iteration constant ≥ 2, the analytical results could be computed iteratively from the scheme given by (10). Figure 2 shows the simulation results for the cases with the iteration constants 1, 2 and 3.

Figure 3 shows the variation of percentage localization with time (in seconds) when the iteration constant equal to 1. For the iteration constant equal to 1, the analytical results match extremely well with the simulation results. For iteration constant ≥ 2, the analytical results obtained from (13) gives an asymptotically tight upper bound for the simulation results. Figure 4 shows the simulation results for the cases with the iteration constants 1, 2 and 3.

8. CONCLUSION

We considered the In-Range localization algorithm in presence of a Mobile Beacon guided by the RWP mobility model and obtained mathematical models for the localization parameters: localization error and percentage localization. The algorithm proposed in this paper uses the concept of Mobile Beacon there by eliminating the need to deploy static beacons in the sensor network. RWP is an easy to implement mobility model in the real world and the In-Range localization scheme relies only on a basic communication capability of the sensors and does not involve any ranging techniques. This paper also attempts to provoke the research in developing mobility models that work well with In-Range localization.

9. FUTURE RESEARCH WORK

Our future research work would concentrate on extending the proposed work to planar sensor networks. The future work would also concentrate on developing and mathematically modeling a heuristics based mobility model for In-Range localization. Research work has to be done on identifying and mathematical modeling of new localization parameters that can act as performance metrics for localization algorithms.

10. PROOFS

Proof of Proposition 5.1: Recall from the definition of Mobile Beacon cardinality that at any given instant of time t, $M(t)$ virtual beacons would be generated by the Mobile Beacon. By using RWP mobility model, these $M(t)$ virtual beacons would follow the uniform probability distribution function in $[-\frac{A}{2}, \frac{A}{2}]$.

It is intuitive that, $\Delta_o^r(t, 1)$ is decided by the left most beacon in the transmission range of o. If there are no beacons in the transmission range of o then $\Delta_o^r(t, 1)$ would be $\frac{A}{2}$ which happens with a probability of $1 - (\frac{2R_0}{A})^{M(t)}$. If there are beacons in the transmission range, then $\Delta_o^r(t, 1)$ would be between 0 and $2R_0$. Let $0 < y \leq 2R_0$, the probability that $\Delta_o^r(t, 1)$ assumes a value $\leq y$ is given by the probability of finding a beacon in the range $[-R_0, -R_0 + y]$, which happens with a probability of $[1-(1 - \frac{y}{A})^{M(t)}]$. Hence the proof.

Proof of Proposition 5.3: At any given instant of time t and iteration constant n, the upper bound for the probability that the tagged node, o is localized is given by the probability of finding at least one beacon in the range $[-nR_0, nR_0]$. The proof follows from the previous statement when applied for RWP mobility model case.

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