Analysis of Raised Cosine Filtering in Communication Systems

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***Abstract*-In this era of wireless communication systems Inter Symbol Interference (ISI) is a major issue. In this paper the inter symbol interference rejection capability of a Raised Cosine Filter (RCF) is analyzed. Simulation is done for analyzing the importance of Rolloff factor and its selection in a raised cosine filter for evaluating the desired response. Communication toolbox in CAD Tool MATLAB is being employed for the simulation.**

I. INTRODUCTION

In a digital transmission system, distortion of the received signal due to intersymbol interference, which is manifested in the temporal spreading of pulses, is an issue of concern for effective signal reception. The pulses overlap to an extent that the receiver cannot reliably distinguish between changes of the state i.e., between individual signal elements.

The raised cosine filter is a particular electronic filter, frequently employed in telecommunication system due to its ability to minimize ISI. Its name stems from the fact that the non-zero portion of the frequency spectrum of its simplest form, having rolloff factor r=1 is a cosine function, which is raised [1, 2]. When used to filter a symbol stream, a Nyquist filter has the property of eliminating ISI, as its impulse response is zero at all nT (where n is an integer), except n=0. Therefore, if the transmitted waveform is correctly sampled at the receiver, the original symbol value can be recovered completely. However, in most practical communication systems, a match filter must be used in the receiver, due to the effect of white noise. This enforces the following constraint [3-4].

$$H_R(f) = H_T{}^*(f) = |H_T(f)| \tag{1}$$

To satisfy this constraint whilst still providing zero ISI, a square root raised cosine filter is typically used at each end of the communication system. In this way, the total response of the system is raised cosine. For designing the raised cosine filter, finite duration impulse response or finite impulse response (FIR) filter is employed. FIR filter have primary advantages as, can have exactly linear phase, always stable, design methods are generally linear, can be realized efficiently in hardware and the startup transient have finite duration. The cost paid for the merits is, that they often require a much higher filter order than IIR filter to achieve a given level of performance. Correspondingly, the delay of these filters is often much greater than for an equal performance IIR filters [2,6].

The paper has four sections as follows; Section I deals with brief introduction. Characteristics of the filter to be designed and simulated are discussed in section II. Simulation and result analysis are part of section III. Section IV deals with conclusion and the references are present at the end of the paper.

II. FILTER MODEL

The filter model prepared for simulation and analysis is based upon the mathematics involved in the Filter design, relative nature of the input to be processed and the desired response to be obtained.

The raised cosine filter is an implementation of a low pass Nyquist filter, i.e. one that has the property of the vestigial symmetry. This means that its spectrum exhibits odd symmetry about 1/2T, where T is the symbol period of the communication system. Its frequency domain description is a piecewise function, given by [1, 3]

$$H(f)=\begin{cases} 1.0, & |f|\le\frac{1-r}{2T} \\ \frac{1}{2}\left[1+\cos\left(\frac{\pi T}{r}\left[|f|-\frac{1-r}{2T}\right]\right)\right], & \frac{1-r}{2T}<|f|\le\frac{1+r}{2T} \\ 0, & otherwise \quad 0\le r\le 1 \end{cases} \tag{2}$$

where r is the rolloff factor

2.1. Characteristics of the Filter

The impulse response of a normal raised cosine filter with rolloff factor r and symbol period T is [1, 2, 4]

$$h(t)=\frac{\sin\left(\frac{\pi t}{T}\right)}{\left(\frac{\pi t}{T}\right)}\times\frac{\cos\left(\frac{\pi rt}{T}\right)}{\left(1-\frac{4r^2t^2}{T^2}\right)} \tag{3}$$

The impulse response of a square root raised cosine filter with rolloff factor r is

$$h(t) = 4r \frac{\cos\left((1+r)\frac{\pi t}{T}\right) + \sin\left(\frac{(1-r)\frac{\pi t}{T}}{4r\frac{t}{T}}\right)}{\pi\sqrt{T}\left(1-\left(4r\frac{t}{T}\right)^2\right)} \quad (4)$$

The impulse response of a square root raised cosine filter convolved with itself is approximately equal to the impulse response of a normal raised cosine filter.

A model developed for simulation should consider the key factor of a raised cosine filter such as, Input and output signals, Noncausality and group delay, Rolloff factor and Filter gain [1, 3-5]. *Input and output signals:* If the input is a sample-based scalar, then the output is a sample-based scalar and the output sample time is N times the input sample time. If the input is frame-based, then the output is a frame-based vector whose length is N (upsampling parameter) times the length of the input vector. The output frame period equals the input frame period. *Noncausality and the group delay parameter:* Without propagation a delay, raised cosine filters are noncausal. This means that the current output depends on the systems future input. In order to design only realizable filters, MATLAB communication Tool, rcosine, and rcosflt functions are used to delay the input signal before producing an output. This delay, known as the filter group delay, is the time between the filters initial response and its peak response. The group delay is defined as

$$\text{Group delay} = -\left(\frac{d\theta(\omega)}{d\omega}\right) \quad (5)$$

where θ is the phase of the filter and ω is the frequency in radian. This delay is set so that the impulse response before time T= 0, is negligible and can safely be ignored by the function. The group delay and the upsampling factor, N determine the length of the filters impulse response, which is (2 * N * Group delay + 1). *Rolloff factor:* This parameter has an important roll in obtaining the desired response at the output. It must be a real number between 0 and 1. The rolloff factor determines the excess bandwidth of the filter. For example, a rolloff factor of 0.5 means that the bandwidth of the filter is 1.5 times the input sampling frequency [1,4]. *The Filter gain:* This is a filter coefficient normalizing parameter. If Filter type is Normal, then the block normalizes the filter coefficients so that the peak coefficient equals 1. If Filter type is Square root, then the block normalizes the filter coefficients so that the convolution of the filter with itself produces a normal raised cosine filter whose peak coefficient equals 1 [1,3].

III. Simulation and Result Analysis

This section is divided into three parts A, B and C. For all the three parts the assumptions made are as follows:
Data length D_L = 20; Digital input signal sampling frequency F_D = 1; Sampling frequency for the filter F_S = 8; Delay (a positive integer) D = 3; Actual group delay or propagation delay in the filter design is D/F_D seconds $Prop_D$ = 8; Rolloff factor (determines the excess bandwidth of the filter) r = 0.5

Part A: Generating, processing and receiving a digital data sequence

Part B: Impact of rolloff factor

Part C: Impact of input sampling frequency factor F_D

Part A. Generating, processing and receiving a digital data sequence

A digital data sequence is generated as per the assumptions above; it is up sampled and filtered using a raised cosine filter as shown in figure 1. It is difficult to compare the digital data sequence and the up sampled filtered output, this is because the peak response of the filter is delayed by the group delay of the filter. If the input signal is delayed by an amount equal to the group delay of the raised cosine filter then the up sampled filtered signal at the output of raised cosine filter is identical to the delayed input signal at the input sampling time as shown in figure 2. This shows the raised cosine filter capability to band limit the signal and over come ISI. In order to get the exact replica of the transmitted signal at the receiver side, the raised cosine filter is splitted between transmitter and receiver. The digital data sequence is up sampled and filtered at the transmitter using the square root raised cosine filter as shown in figure 3. The transmitted signal is filtered at the receiver side using the square root raised cosine filter (without up sampling). The resulting signal at the output is identical to the signal filtered using a single raised cosine filter (as shown in figure 3.), i.e. effective recovery of the signal is possible.

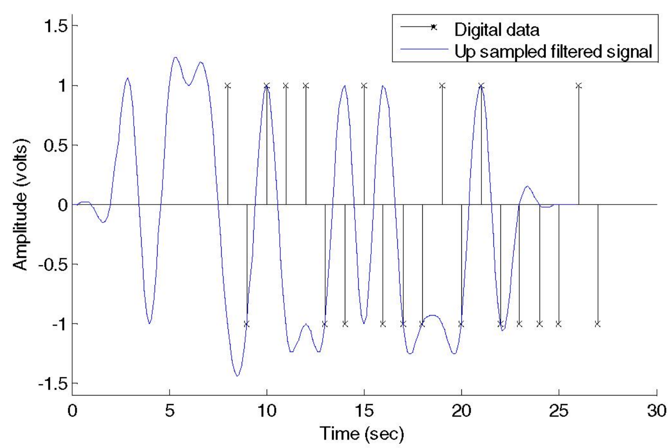

Figure 1. Comparison of digital data sequence with the up sampled filtered signal.

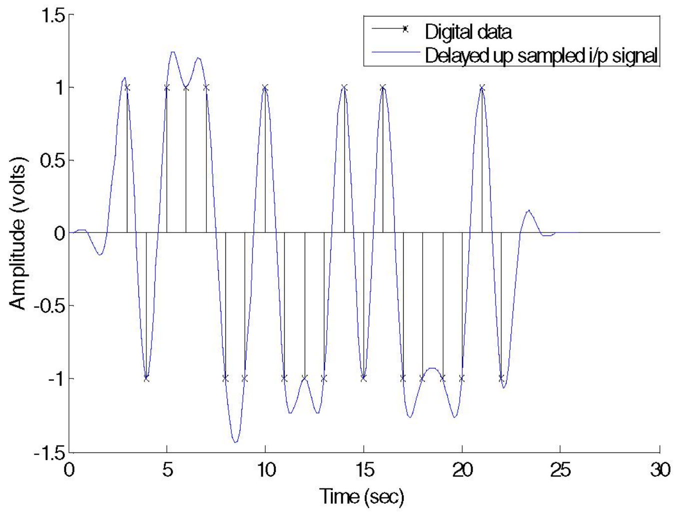

Figure 2. Comparison of digital data sequence with the delayed i/p up sampled filtered signal

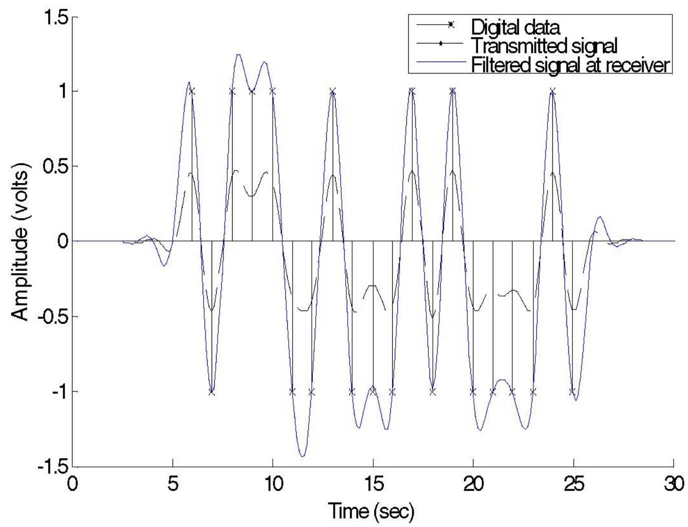

Figure 3. Plot showing the digital data sequence, the transmitted signal and the signal after filtration at the receiver.

Part B. Impact of rolloff factor

Keeping all the parameters same as per the assumption and changing the value of rolloff factor to 0.2, 0.5 and 0.7. If the rolloff factor is decreased from value 0.5 to 0.2 peak overshoot of the filtered signal is observed as shown in figure 4. The lower value of the roll of factor causes the filter to have a narrow transition band causing the filtered signal overshoot. On the other hand, if the rolloff factor is increased from 0.5 to 0.7 or above, a low peak overshoot of filtered signal is observed as shown in figure 5. This is due to the increase in the transition band of the filter. Effective signal recovery is observed, when the rolloff factor has value 0.5.

Part C. Impact of input sampling frequency factor F_D

If the factor F_D is decreased from 1 to 0.5, a mismatch of the digital data sequence with the filtered output signal is observed as shown in figure 6. It may be further noted that filtered output signal is a head of the digital data sequence due to which exact signal recovery is difficult. On the other hand if the factor F_D is increased from 1 to 2, again proper mismatch of the digital data sequence and the filtered output at the receiver occur, where the digital data sequence is a head of the filtered output signal as shown in figure 7. If the factor F_D is further increased from 2 to 2.5, filtered output signal is not observed, i.e. signal cannot be recovered.

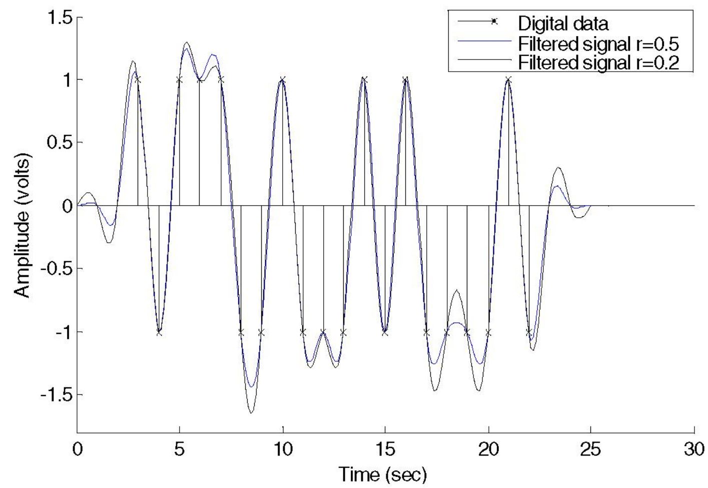

Figure 4. Digital data sequence and the signal after filtration at the receiver for filter rolloff factor r = 0.2 and 0.5

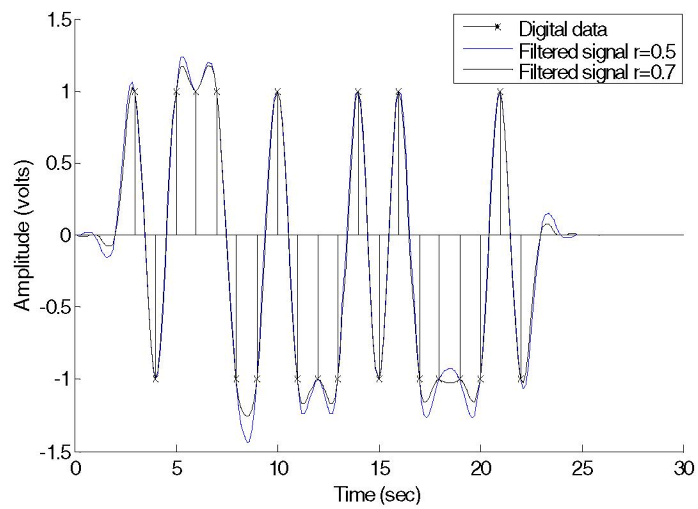

Figure 5. Digital data sequence and the signal after filtration at the receiver for filter rolloff factor r = 0.7 and 0.5

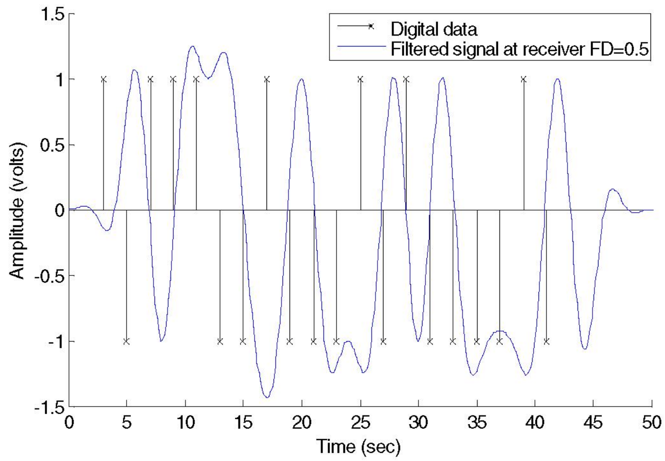

Figure 6. Digital data sequence and the signal after filtration at the receiver for $F_D = 0.5$

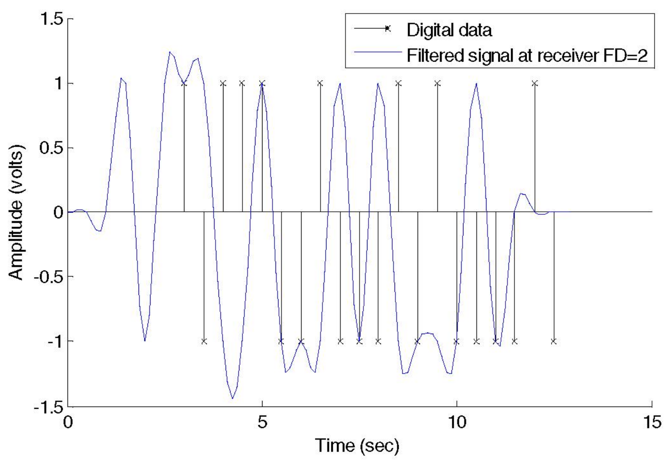

Figure 7. Digital data sequence and the signal after filtration at the receiver at $F_D = 2$

IV. CONCLUSION

In any communication system employing digital transmission of data, signal can be effectively recovered, if proper value of different parameters such as input signal sampling frequency factor, Rolloff factor and Group delay factor are selected. If the input signal is delayed by an amount equal to the group delay of the raised cosine filter then the up sampled filtered signal at the output of raised cosine filter is identical to the delayed input signal at the input sampling time. Peak overshoot occurs if the rolloff factor is decreased from 0.5 to 0.2. On the other hand, an increase in the rolloff factor lead to mismatch of the digital data transmitted and the filtered output signal at the receiver. Group delay and the rolloff factor further helps to reduce the inter symbol interference. Recovery of the transmitted digital data is also affected if the input signal sampling frequency factor is not properly selected.

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