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Hidden Markov Model-Contourlet Hidden Markov Tree Based Texture Segmentation

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Abstract- **Contourlets have emerged as a new mathematical tool for image processing and provide compact and decorrelated image representations. Hidden Markov modeling (HMM) of contourlet coefficients is a powerful approach for statistical processing of natural images. In this paper, we extended the hidden Markov modeling framework to contourlets and combined hidden Markov trees (HMT) with hidden Markov model to form HMM-Contourlet HMT model. The model is used for block based multiresolution texture segmentation. The performance of the HMM-Contourlet HMT texture segmentation method is compared with that of HMM-Real HMT and HMM-Complex HMT methods. The HMM-Contourlet HMT method provides superior texture segmentation results and excellent visual performance at small block sizes.**

I. INTRODUCTION

In statistical image segmentation, it is necessary to capture both global and local statistical structure of textures. Such block-based modeling considering statistical dependencies between blocks can result in a better segmentation. The contourlet transform is better suited for representing singularities such as edges and ridges in an image that characterize textures. The multiscale-multiresolution property of contourlets makes HMM based texture segmentation possible.

The HMM framework for real discrete wavelet transform suffers from shift variance that degrades the accuracy of the segmentation results. The real wavelet gives only three directional features and fails to give information at diagonal orientations. The authors in [1, 2] described HMT modeling for wavelet coefficients of natural image. The model is used for image segmentation in [3]. In [4] HMM-Real HMT modeling is presented for texture segmentation achieving improved segmentation performance.

In [5] dual tree complex wavelet transform is developed which is approximately shift invariant and has improved directional selectivity compared to the real wavelet transforms. The segmentation performance based on this complex wavelets in [6, 7] is still improved. But this complex wavelet is having fixed six directional subbands and processing of coefficients is computationally intensive even though only magnitude is considered.

Apart forms this, the contourlets provide image representations at varying directions in multiple scales. One can select the number of directional orientations and scales for image decomposition. In this paper, we combined hidden Markov trees and hidden Markov model to form HMM-Contourlet HMT model for contourlet baesd multiscale texture segmentation.

II. RELATED WORK

A. Contourlet Transform

The contourlet transform is a new extension to the wavelet transform in two dimensions [8]. This uses non-separable and directional filter banks. The basis images are oriented at varying directions. With this rich set of basis images, the contourlet transform can efficiently capture the smooth contours in the natural texture images. The geometrical structures are well captured by the contourlets. The contourlets possess multiresolution, time-frequency localization and high degree of directionality.

The contourlets are implemented using pyramidal directional filter banks (PDFB) [8, 9]. This PDFB is a cascade of Laplacian pyramid and a directional filter bank. The directional filter bank can decompose an image into any power of two's number of directions. The multiscale and directional decompositions are independent of each other. Fig. 1 shows two-scale, four directional contourlet decomposition of Peppers image.

The contourlet basis functions have rectangular support regions. Edges in this region affect the magnitude of coeffficients. The edges that are picked out by the contourlets contribute energy to a small number of coefficients. The coefficients for natural images exhibit residual dependency structure both across and within scale. Images have been modeled based on these dependencies.

The contourlet coefficients of 2-D images are arranged in the form of quad-trees [9]. A coefficient in a low subband (parent) can be thought of as having four descendants (children) in the next higher subband. The four descendants

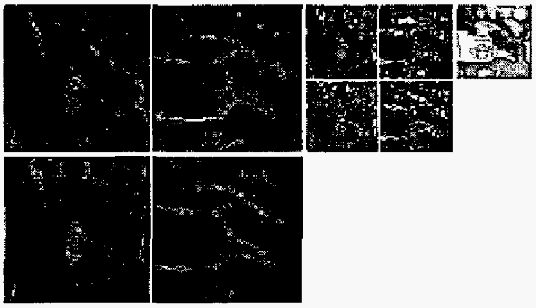


Fig. 1. Contourlet decomposition of Peppers image

each also have four descendants in the next higher subband and a quad tree will emerge. Thus, the coefficients are represented in quad trees and inter scale dependencies are captured using HMM.

Fig. 2 shows the histogram of the finest subband of contourlet coefficients of an image. The distribution is characterized by a sharp peak at zero amplitude and extended tails on either side of the peak. This implies that the contourlet transform is very sparse. Thus the marginal distributions of natural images in the contourlet domain are highly non-Gaussian. The large/small values of the coefficients tend to propagate across scale that shows coefficients persistence property.

B. *Contourlet HMT model*

The HMT model is used to model the joint pdf of contourlet coefficients of an image. The marginal pdf of each coefficient is modeled as a Gaussian mixture density with unobserved hidden states. Modeling includes two stages: modeling the marginal density of each wavelet coefficient and modeling the dependencies between the coefficients.

Each coefficient c_i is associated with a set of discrete hidden states $S_i = 0,1,...,M-1$ (for M state model) which have probability mass function (pmf) $p_{S_i}(m)$. Given $S_i = m$, the pdf of the coefficient c is Gaussian with mean μ_m and variance σ_m^2. The Gaussian distribution with mean μ, variance σ^2, can be written as:

$$g(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\} \qquad (1)$$

The parameter vector of a M state Gaussian Mixture Model (GMM) is

$$\pi = \{\ p_{S_i}(m), \mu_m, \sigma_m^2 \big| m = 0,1,..., M-1\ \} \qquad (2)$$

and the over all pdf of C is determined by the sum

$$f_C(c) = \sum_{m=0}^{M-1} p_{S_i}(m) f_{C|S_i}(c \big| S_i = m) \qquad (3)$$

$$f_{C|S_i}(c \big| S_i = m) = g(c;\mu_m,\sigma_m^2) \qquad (4)$$

Consider a two state GMM, where each coefficient c_i is associated with a hidden state S_i taking value 0 and 1,

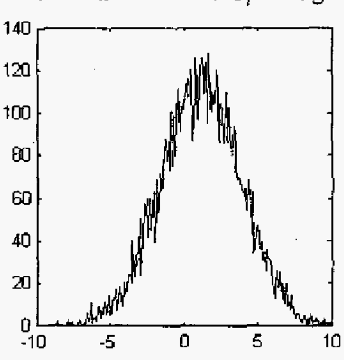


Fig. 2. Histogram of contourlet coefficients

depending on c_i is small or large respectively. The state variable S_i tells that, from which of the two components in the mixture model c_i is drawn. Each coefficient c_i is conditionally Gaussian given its state variable S_i.

The state 0 corresponds to a low variance Gaussian with pdf $f(c_i \big| S_i = 0) = g(c_i;\mu_i,\sigma_{S,i}^2)$ and the state 1 corresponds to a high variance Gaussian with pdf $f(c_i \big| S_i = 1) = g(c_i;\mu_i,\sigma_{L,i}^2)$. Note that $\sigma_L^2 > \sigma_S^2$. The marginal pdf is obtained by taking sum of the conditional densities as;

$$f(c_i) = p_i^S g(c_i;\mu_i,\sigma_{S,i}^2) + p_i^L g(c_i;\mu_i,\sigma_{L,i}^2) \qquad (5)$$

$$p_i^S + p_i^L = 1 \qquad (6)$$

where, p_i^S and p_i^L are state value pmfs for S_i={0,1} respectively and can be interpreted as the probability that c_i is small or large respectively. A smooth region of the image is captured by S_i=0 and edge region is captured by S_i=1. Even though the contourlet transform gives uncorrelated coefficients, there exists considerable amount of high-order dependencies. Hence coefficients are statistically dependent along the branches of the tree.

The expected magnitude of a contourlet coefficient is closely related to the size of its parent. This implies a type of Markovian relationship between the contourlet states, with the probability of a contourlet coefficient being large affected by the size of its parent. This makes the state of the children coefficients depend on the state of the parent. The dependence is modeled as Markov-1. In the HMM, these dependencies across the scale are captured using a probabilistic tree that connects the hidden state variable of each coefficient with the state variable of each of its children.

Each subband is represented with its own quad-tree; thus quad trees are assumed independent. The dependencies across the scale (between each parent and its children) form the transition probabilities between the hidden states. The transitions among the states are governed by a set of transition probabilities. The 0 to 0 and 1 to 1 transitions have higher probabilities due to the persistence of contourlet coefficients.

The parameter $\varepsilon_{j,j-1}^{m,n} = p(S_i = n \big| S_{p(i)} = m)$ gives the probability that a child coefficient is in a hidden state n, when its parent $c_{p(i)}$ is in state m, where $p(i)$ is the parent of node i and scale $j = 1,...,J-1$ (J is the coarsest scale), $m,n = 0,1$. Each parent to child state-to-state link (transition probabilities) has a corresponding state transition matrix

$$A_i = \begin{bmatrix} p_i^{0\to 0} & p_i^{0\to 1} \\ p_i^{1\to 0} & p_i^{1\to 1} \end{bmatrix} \qquad (7)$$

with $p_i^{0\to 1} = 1 - p_i^{0\to 0}$ and $p_i^{1\to 0} = 1 - p_i^{1\to 1}$. The matrix has the row sums equal to unity. The parameters $p_i^{0\to 0}$ / $p_i^{1\to 1}$ are the probability that contourlet coefficient c_i is small/large given that its parent is small/large. These are the persistency probabilities. The parameters $p_i^{1\to 0}$ and $p_i^{0\to 1}$ are the probabilities that the state values will change from one scale to the next. To propagate the large and small coefficient values down the quad-tree it is required that, $p_i^{0\to 0} > p_i^{0\to 1}$ and $p_i^{1\to 1} > p_i^{1\to 0}$.

Contourlet HMT Parameters are

1) Gaussian mixture means $\mu_{i,m}$ and variances $\sigma_{i,m}^2$.
2) The transition probabilities $p(S_i | S_{p(i)}) = \varepsilon_{j,j-1}^{m,n}$.
3) The pmf of the root node $p_0(m)$ in the coarsest scale.

The parameters vector is represented as; $\theta = \{p_{S_0}(m), \varepsilon_{j,j-1}^{m,n}, \mu_{i,m}, \sigma_{i,m}^2 | m, n = 0,1\}$. The HMM is trained to capture the contourlet domain features of the image of interest using the iterative expectation maximization (EM) algorithm.

III. HMM-Contourlet HMT Model

Consider a texture image and divide the image into blocks of size $2^M \times 2^M$ where, M is the level of decomposition. Consider the nine contiguous blocks of the texture image. It is assumed that all such blocks are associated with the same texture. Each block is associated with a hidden state and may occupy a different texture dependent state. The state of a given block is linked to the state of the eight surrounding contiguous blocks. The dependency between the blocks shown in Fig. 3 is Markov-I. A given $2^M \times 2^M$ block is assumed to reside in a particular state of a certain texture. The statistical likelihood of such a state is dictated by the states occupied by the adjoining blocks[10]. Inter block statistics are modeled by HMM and intra-block state-dependent contourlet statistics are modeled using HMT. We denote this as HMM-Contourlet HMT model.

For a given texture type T_x, a set of textural states S_l is defined. The set S_l represents the l^{th} state of the texture. Let c_q represent the contourlet coefficients associated with the four quad trees where, $q = \{1, 2, 3, 4\}$. The corresponding

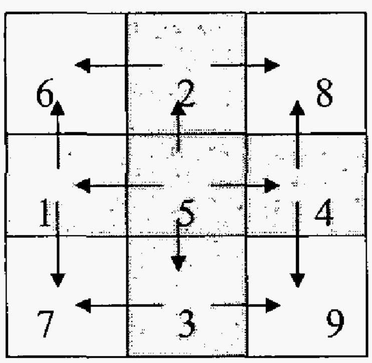


Fig. 3. Eight neighborhood system of blocks

HMTs are $p(c_q | S_l)$.

For modeling we have considered four directional subbands of contourlet coefficients. Let c^j represent the contourlet coefficients associated with block j, $1 \le j \le 9$ and $c^j = \{c_1^j, c_2^j, c_3^j, c_4^j\}$. The conditional likelihood of c^j from the parameters of the corresponding HMT is $p(c^j | S_j)$.

The likelihood that the coefficients c^j are associated with texture T_x is given by

$$p(c^j | T_x) = \sum_{k1=1}^{K} \cdots \sum_{k9=1}^{K} p(S_{k1}^1 \ldots S_{k9}^9) p(c^1 | S_{k1}^1) \ldots p(c^9 | S_{k9}^9) \quad (8)$$

The texture is characterized by K states. The likelihood of the states is calculated as follows.

$$p(S_{k1}^1 \ldots S_{k9}^9) = p(S^5 | S^j) p(S^j) \qquad \forall j \ne 5 \quad (9)$$

$$= p(S^5 | S^j) p(S^1, S^2, S^3, S^4) p(S^6, S^7, S^8, S^9 | S^1, S^2, S^3, S^4)$$

It is assumed that the statistical dependence between adjacent blocks will be strong for those blocks sharing a common edge and blocks not sharing a common edge are approximately independent. Then the likelihood of state is

$$p(S_{k1}^1 \ldots S_{k9}^9) = p(S^5)\ [p(S^1 | S^5) p(S^2 | S^5) p(S^3 | S^5) p(S^4 | S^5)] \times$$

$$[p(S^6 | S^1, S^2) p(S^7 | S^1, S^3) p(S^8 | S^2, S^4) p(S^9 | S^3, S^4)] \quad (10)$$

Consider the training of texture images. Assume that the texture is characterized by k states $\{S_k,\ 1 \le k \le K\}$. A distinct set of training blocks is assigned to each of the K states. EM algorithm is used for training HMTs associated with quad tree contourlet coefficients. Inter block HMM is trained as follows. For the texture, let π_k represent the probability that the block 5 is in S_k. $\sum_k \pi_k = 1$. These form the initial state probabilities.

$$\pi_k = p(S^5 = k) \quad k = 1, \ldots, K \quad (11)$$

The probability of transitioning from state S_k in block 5 to S_l in block 1 is α_{kl} for $1 \le k \le K$ and $1 \le l \le K$. $\sum_l \alpha_{kl} = 1$. State transitions from block 5 to blocks 1 through 4 are approximated as independent and identical. So the probability of transitioning from S_k in block 5 to S_l in block 2 is also α_{kl}. The probability that block 6 is in S_m given that block 1 and 2 are in S_k and S_l respectively is β_{kl}^m. $\sum_m \beta_{kl}^m = 1$. The same transition probabilities are also used to model transitions to blocks 7 through 9. Let $b_k(c^j)$ represent the likelihood that the contourlet quad tree coefficients c^j is from block j.

$$b_k(c^j) = p(C^j | S^j = k) \quad (12)$$

The Markov model for the sequence of states of nine contiguous blocks is shown in Fig. 4. Parameters for this model are estimated by the combination of a modified Viterbi

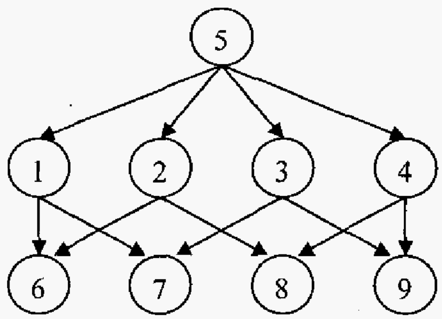


Fig. 4. Hidden Markov model for nine blocks

algorithm for the global HMM part and an EM algorithm for the local HMT part.

IV. Texture Segmentation

In supervised texture segmentation, before training it is necessary to acquire training data representative of each texture for the models. Training images are obtained either by taking homogeneous regions of the given image or from completely different images having homogeneous regions representative of the candidate textures. From each class $cl = \{1,..., N_c\}$, it is necessary to train the model.

HMM-Contourlet HMT model is developed for each texture of interest based on training data. The middle block (block 5) is assigned to that texture for which the associated HMM-Contourlet HMT model yields a maximum likelihood. Rather than segmenting all nine blocks to a given texture, only middle block is so segmented. Each of the blocks in the image sequentially plays the role of block 5, there by yielding segmentation. Those eight blocks with which it has direct contact influence the texture assigned to a given block. According to the HMM assumption, each image block is independent of the rest of the image given eight surrounding blocks.

Having trained the model for each class i of textures, let the resulting set of model parameters be θ_i. The likelihood of the block 5 to be a class i is

$$p(block\ \ 5|\theta_i) = \max_{S_{k1}^1,...,S_{k9}^9} \left\{ \begin{array}{l} p(S_{k1}^1,...,S_{k9}^9|\theta_i)\times \\ \qquad p(c^1|S_{k1}^1,\theta_i)...p(c^9|S_{k9}^9,\theta_i) \end{array} \right\}$$

In maximum likelihood segmentation, the class label that maximizes this likelihood is assigned to the block.

$$i_{ML} = \arg\max_i \quad p(block\ \ 5|\theta_i) \qquad (14)$$

Let $L_i = p(C^j|T_i)$ be the likelihood. Block 5 is assigned to that texture T_k for which $L_k > L_i$ for all textures $T_i \neq T_k$. In testing phase, consider the contourlet coefficients $\tilde{c}$ of an image containing montage of these textures. The likelihood is calculated for each block. The block to be assigned to a particular texture plays the role of block 5. For each class i of textures, from the set of parameters θ_i trained, compute the likelihood

$$p(block\ \ 5|\theta_i) = \max_{S_{k1}^1,...,S_{k9}^9} \left\{ \begin{array}{l} p(S_{k1}^1,...,S_{k9}^9|\theta_i)\times \\ \qquad p(\tilde{c}^1|S_{k1}^1,\theta_i)...p(\tilde{c}^9|S_{k9}^9,\theta_i) \end{array} \right\}$$

$$\pi_k^i = p(S^5 = k|\theta_i) \qquad k = 1,...,K \qquad (16)$$

$$b_k^i(\tilde{c}^j) = p(\tilde{c}^j|S^j = k,\theta_i) \qquad (17)$$

where, $\tilde{c}^j$ is for the test image. Assign the class label that maximizes at each scale.

In the training phase, first the number of states is selected. HMM and associated HMTs are initialized. Hidden states of HMM are evaluated. HMTs are reestimated based on the states of HMM. State transition probability of HMM is reestimated. This reestimation procedure is carried out to get the maximum likelihood (ML) model parameters.

V. Test Results

We used Haar filter for both directional and multiscale decompositions and the HMM-Contourlet HMT model is trained for each training texture image using only two-state model ($K = 2$). Three level contourlet decomposition of image is used. The maximum block size is set to 4×4. The blocks are divided into two groups according to their estimated states.

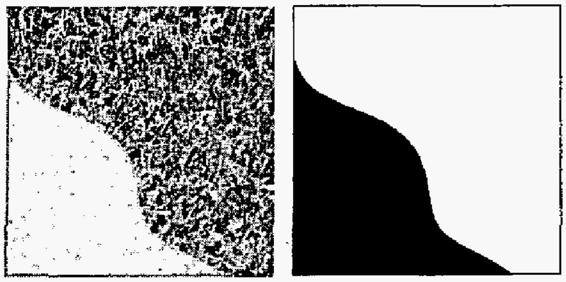
The block states are initialized as follows. The median pixel value md is obtained from the training texture. For each block, if the block mean is greater than md, the block is set to state 1; otherwise the block is set to state 2. The block states are reestimated iteratively and hence the group of HMTs associated with a state varies per iteration.

Experiments are done for synthetic two-texture images, in which ground truth is clear. The image shown in Fig. 5 (a) is a montage of Grass Texture and Water Texture from the USC SIPI image database [11]. From the 512×512 original image, we used a one-forth for training image and another one-forth to make a test image. Results are shown for the block sizes of 2×2 and 4×4. The segmented images of different block sizes are obtained simultaneously, realizing multiresolution segmentation.

The models taking global dependency into account shows better performance. The HMM-Contourlet HMT shows a much homogeneous segmentation result than the HMM-Real HMT. The quality of segmentation is compared by calculating average percentage segmentation error obtained from the pixel by pixel comparison with the ground truth. The Table I shows the performance of different segmentation methods. The texture segmentation results for different methods are depicted in Fig. 6, Fig. 7, and Fig. 8.

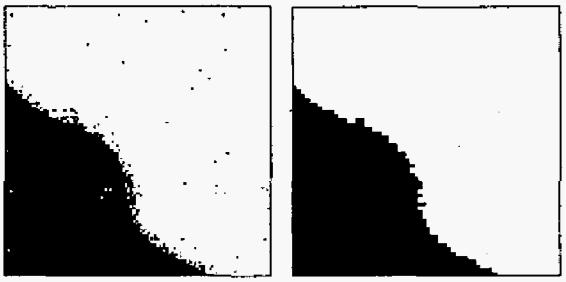
VI. Conclusion

In this paper we presented a multiscale-multiresolution block based texture segmentation algorithm by modeling contourlet coefficients using HMM-Contourlet HMT model.



(a) (b)

Fig. 5. (a) Test image (b) Ground truth



(a) (b)

Fig. 6. Texture segmentation using HMM-Real HMT method
(a) 2x2 block (b) 4x4 block.

(a) (b)

Fig. 7. Texture segmentation using HMM-Complex HMT method
(a) 2x2 block (b) 4x4 block.

(a) (b)

Fig. 8. Texture segmentation using HMM-Contourlet HMT method
(a) 2x2 block (b) 4x4 block.

TABLE I
AVERAGE PERCENTAGE ERROR IN SEGMENTATION

Texture Segmentation Method	Block Size	
	2x2	4x4
HMM-Real HMT	2.69	3.20
HMM-Complex HMT	2.55	3.59
HMM-Contourlet HMT	1.93	4.16

The HMM-Contourlet HMT model takes care of both the global and local statistics in block based segmentation. It can segment contourlet-transformed data directly without re-transforming to the space domain.

The performance of the HMM-Contourlet HMT method is compared with that of HMM-Real HMT and HMM-Complex HMT methods. The average error in the contourlet based segmentation is less for small block sizes when compared with that of the other two methods. The experimental performance comparison shows that HMM–Contourlet HMT segmentation algorithm is superior for small block sizes and gives excellent visual segmentation results.

The algorithm based on the contourlets provides raw segmentation results that can be used as a front end in a more sophisticated multiscale segmentation algorithms based on inter scale fusion. The performance evaluation of HMM-Contourlet HMT algorithm for segmenting synthetic images, aerial photos, document images radar/sonar images, medical images and multidimensional data in geophysical surveys is an interesting future work.

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